University of Global Village (UGV), Barishal
Dept. of Electrical and Electronic Engineering (EEE)

RF & Microwave
EEE 321

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'Imagination is more important than knowledge' - Albert Einstein

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Basic Course Information

Course Title	RF & Microwave
Course Code	EEE-321
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	6th Semester
Academic Session	Winter 2025

Electrical Circuit I Sessional (EEE 0713-1102)

3 Credit Course

Class:	**17 weeks (1 classes per week)** **Total Class Duration: 2 hrs.** **Total Practice Duration: 3 hrs.** **Total=85 Hours**
Preparation Leave (PL):	02 weeks
Exam:	04 weeks
Results:	02 weeks
Total:	**25 Weeks**

Attendance:

Students with more than or equal to 70% attendance in this course will be eligible to sit for the Semester End Examination (SEE). SEE is mandatory for all students.

Continuous Assessment Strategy

Quizzes

Altogether 4 quizzes may be taken during the semester, 2 quizzes will be taken for midterm and 2 quizzes will be taken for final term.

Assignment

Altogether 2 assignments may be taken during the semester, 1 assignments will be taken for midterm and 1 assignments will be taken for final term.

Presentation

The students will have to form a group of maximum 3 members. The topic of the presentation will be given to each group and students will have to do the group presentation on the given topic.

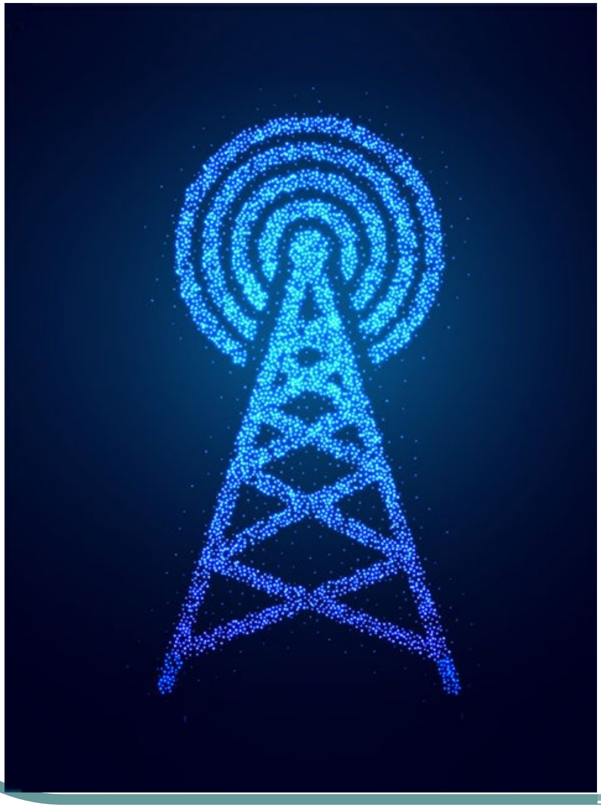
ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)			
Bloom's Category Marks	Tests (45)	Quiz (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10	09	Bloom's Affective Domain: (Attitude or will) Attendance: 15 Viva-Voca: 5 Assignment: 5 Presentation: 5
Understand	8	06	
Apply	10		
Analyze	5		
Evaluate	7		
Create	5		

SEE- Semester End Examination (60 Marks)	
Bloom's Category	Tests
Remember	**10**
Understand	**10**
Apply	**15**
Analyze	**10**
Evaluate	**10**
Create	**5**

Course Learning Outcome (CLO)

Course learning outcomes (CLO): After successful completion of the course students will be able to -



CLO-1 **Understand** the fundamental principles and characteristics of RF and microwave frequencies.

CLO-2 **Analyze** transmission lines, waveguides, and impedance matching techniques.

CLO-3 **Design** and **evaluate** RF and microwave components, such as filters, amplifiers, and antennas.

CLO-4 **Apply** S-parameters and network analysis to characterize RF circuits.

SYNOPSIS / RATIONALE

RF and Microwave Engineering is essential for modern communication, radar, and wireless systems. This course connects electromagnetic theory with practical high-frequency circuit design, focusing on unique phenomena like transmission line effects and impedance matching. It prepares students to design RF/microwave components and systems, addressing real-world challenges in telecommunications, IoT, aerospace, and defense.

Course Objectives

- To provide an understanding of the fundamental concepts and principles of RF and microwave engineering.
- To equip students with the ability to analyze and design transmission lines, waveguides, and impedance matching networks.
- To develop skills for designing and implementing RF and microwave components such as filters, amplifiers, and antennas.
- To familiarize students with S-parameters, network analysis, and their applications in RF circuit characterization.
- To prepare students to address real-world challenges in RF and microwave systems used in communication, radar, and other advanced technologies.

COURSE OUTLINE

Sl.	Content of Course	Hrs	CLOs
1	**Introduction to Microwaves**: Definition, significance, applications, and limitations of microwaves in communication engineering.	6	CLO1
2	**Electromagnetic Wave Propagation**: Transverse waves, circuit theory limitations, and Telegrapher's equations in time and frequency domains.	8	CLO2
3	**Transmission Line Analysis**: Distributed parameters, attenuation, phase constant, impedance, and Smith Chart for impedance matching.	10	CLO3
4	**Waveguide and Microwave Components**: Waveguide tees, magic tees, directional couplers, and properties of S-parameters.	8	CLO4

COURSE OUTLINE

Sl.	Content of Course	Hrs	CLOs
5	**Microwave Tubes and Semiconductor Devices**: Reflex and two-cavity klystrons, TWTs, diodes, BJTs, TEDs, IMPATT, TRAPATT, and BARITT.	10	CLO5
6	**Microwave Design Techniques**: Impedance matching using short stubs, circulator and coupler design, and real-world applications.	6	CLO3, CLO4
7	**Advanced Applications of RF and Microwave**: Practical applications in communication systems and problem-solving with learned tools and concepts.	4	CLO1 – CLO4

COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Introduction to Microwaves: Basics, why they are called microwaves, and their suitability in communication	Lecture, multimedia presentations	Class participation, Q&A	CLO1
2	Applications of Microwaves and Limitations of High-Frequency Use	Case studies, discussions on real-world applications	Short quiz	CLO1, CLO2
3	Electromagnetic Waves: Transverse waves and circuit theory limitations at high frequencies	Concept reinforcement through problem-solving	Assignment	CLO2
4	Telegrapher’s Equations: Time-domain and frequency-domain derivations	Interactive lecture, numerical exercises	Class Test 1	CLO2, CLO3

COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
5	Transmission Line Characteristics: Attenuation, phase constant, and impedance for various line types	Practical examples, problem-solving	Problem Solve	CLO2, CLO3
6	Impedance Matching: Principles, Smith Chart applications, and stub design	Design-based learning, problem-solving using Smith Chart	Assignment	CLO3, CLO4
7	Reflection Coefficient and VSWR Calculations	Step-by-step calculation examples	Short quiz	CLO3
8	Waveguide Tees and Directional Coupler Basics	Lecture and demonstrations	Class Test 2	CLO4
9	S-Parameters: Properties and Symmetry Analysis	Case studies, problem-solving	Numerical problem	CLO4, CLO3

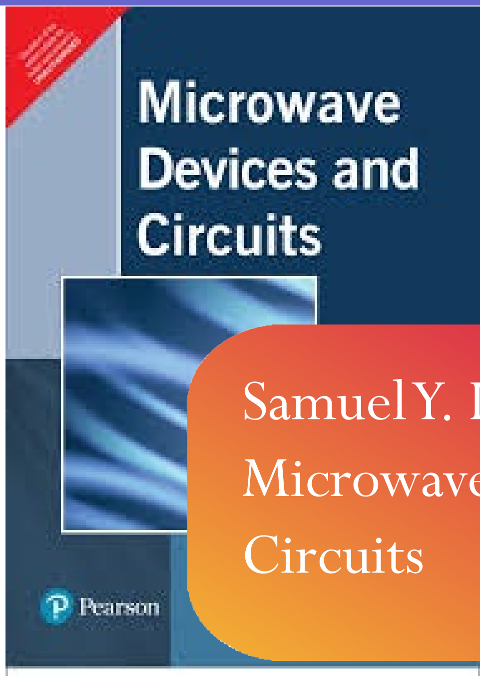
COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
10	Magic Tee and Circulator S-Matrix Analysis	Interactive lectures, application discussions	Assignment	CLO3, CLO4
11	Microwave Tubes: Classification and operating principles of Two-Cavity and Reflex Klystrons	Diagrams, operational explanations	Short quiz	CLO4
12	Traveling Wave Tubes (TWT): Comparison with Klystrons	Lecture, comparative discussions	Class Test 3	CLO4
13	Microwave Semiconductor Devices: Diodes, BJTs, TEDs, and their comparisons	Case studies, diagrams	Assignment	CLO3

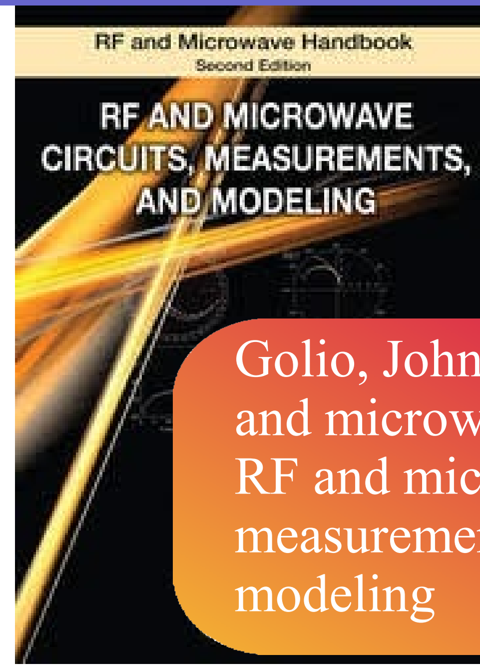
COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
14	Advanced Microwave Components: IMPATT, TRAPATT, and BARITT diodes	Interactive lectures, problem-solving	Numerical problem sets	CLO4
15	Revisiting Core Topics: Problem-solving and reinforcing concepts	Group discussions, practical problem-solving	Review session	CLO1–CLO4
16	Practical Applications of Microwaves in Modern Technologies	Case studies, interactive discussions	Assignment	CLO1–CLO4
17	Final Class Test	Comprehensive evaluation	Class Test 4	CLO1–CLO4
18	Course Feedback and Recap	Summary discussions, reflective Q&A	Feedback	CLO1–CLO4

REFERENCE BOOK



Samuel Y. Liao - Microwave Devices and Circuits



Golio, John Michael -RF and microwave handbook : RF and microwave circuits, measurements, and modeling

Video Lecture Playlist
https://youtube.com/playlist?list=PLhdVEDm7SZObfQ1O1RzBGhqh1NxVDoCW&si=zdIlK56sTaLIO4gW

Bloom Taxonomy Cognitive Domain Action Verbs

Remembering (C1)	Choose • Define • Find • How • Label • List • Match • Name • Omit • Recall • Relate • Select • Show • Spell • Tell • What • When • Where • Which • Who • Why
Understanding (C2)	Classify • Compare • Contrast • Demonstrate • Explain • Extend • Illustrate • Infer • Interpret • Outline • Relate • Rephrase • Show • Summarize • Translate
Applying (C3)	Apply • Build • Choose • Construct • Develop • Experiment with • Identify • Interview • Make use of • Model • Organize • Plan • Select • Solve • Utilize
Analyzing (C4)	Analyze • Assume • Categorize • Classify • Compare • Conclusion • Contrast • Discover • Dissect • Distinguish • Divide • Examine • Function • Inference • Inspect • List • Motive • Relationships • Simplify • Survey • Take part in • Test for • Theme
Evaluating (C5)	Agree • Appraise • Assess • Award • Choose • Compare • Conclude • Criteria • Criticize • Decide • Deduct • Defend • Determine • Disprove • Estimate • Evaluate • Explain • Importance • Influence • Interpret • Judge • Justify • Mark • Measure • Opinion • Perceive • Prioritize • Prove • Rate • Recommend • Rule on • Select • Support • Value
Creating (C6)	Adapt • Build • Change • Choose • Combine • Compile • Compose • Construct • Create • Delete • Design • Develop • Discuss • Elaborate • Estimate • Formulate • Happen • Imagine • Improve • Invent • Make up • Maximize • Minimize • Modify • Original • Originate • Plan • Predict • Propose • Solution • Solve • Suppose • Test • Theory

EEE 4181

MICROWAVE ENGINEERING

PART 1:

TRANSMISSION LINE THEORY

Week 1

Slide 19-30

OUR MENU

- Introduction to Microwaves
- Transmission Line Equations
- The Lossless Line
- Terminated Transmission Lines
  - Reflection Coefficient
  - VSWR
  - Return Loss
- Transmission Line's Impedance Equation
- Special Cases of Terminated Transmission Lines

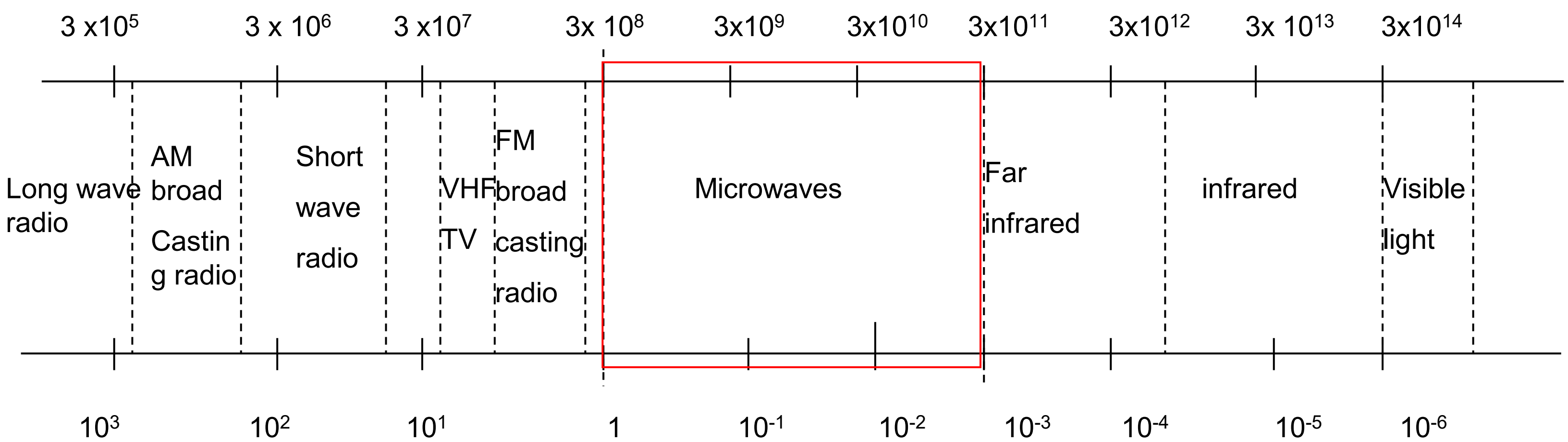
What is microwave?

- Microwave refers to alternating current signals with wavelengths between 1 m and 1 mm (corresponding frequencies between 300 MHz and 300 GHz).

Wavelength of a wave is the distance we have to move along the transmission line for the sinusoidal voltage to repeat its pattern

INTRODUCTION (Contd.)

- Figure 1 shows the location of the microwave frequency



Typical frequencies

AM broadcast band 535-1605 kHz

Shortwave radio 3-30 MHz

FM broadcast band 88-108 MHz

VHF TV (2-4) 54-72 MHz

VHF TV (5-6) 76-88 MHz

UHF TV (7-13) 174-216 MHz

UHF TV (14-83) 470-890 MHz

Microwave ovens 2.45 GHz

SPECTRUM & WAVELENGTHS

Things to Remember

- The higher the frequency, the more energy the wave has.
- EM waves do not require media in which to travel or move.
- EM waves are considered to be transverse waves because they are made of vibrating electric and magnetic fields at right angles to each other, and to the direction the waves are traveling.
- Inverse relationship between wave size and frequency: as wavelengths get smaller, frequencies get higher.

SPECTRUM & WAVELENGTHS

The Waves (in order…)

Radio waves: Have the longest wavelengths and the lowest frequencies; wavelengths range from 1000s of meters to .001 m

- Used in: RADAR, cooking food, satellite transmissions

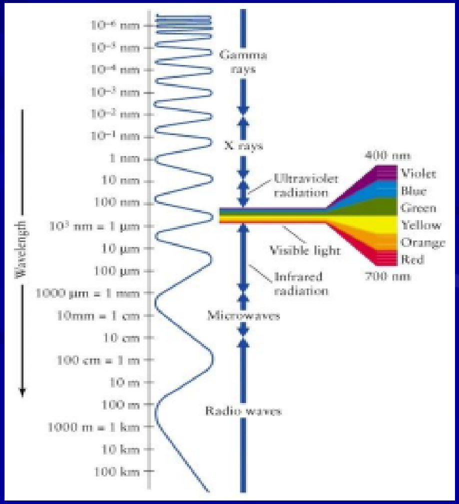
SPECTRUM & WAVELENGTHS

Infrared waves (heat): Have a shorter wavelength, from .001 m to 700 nm, and therefore, a higher frequency.

- Used for finding people in the dark and in TV remote control devices

Visible light: Wavelengths range from 700 nm (red light) to 30 nm (violet light) with frequencies higher than infrared waves.

- These are the waves in the EM spectrum that humans can see.
- Visible light waves are a very small part of the EM spectrum!



SPECTRUM & WAVELENGTHS

Ultraviolet Light: Wavelengths range from 400 nm to 10 nm; the frequency (and therefore the energy) is high enough with UV rays to penetrate living cells and cause them damage.

- Although we cannot see UV light, bees, bats, butterflies, some small rodents and birds can.
- UV on our skin produces vitamin D in our bodies. Too much UV can lead to sunburn and skin cancer. UV rays are easily blocked by clothing.
- Used for sterilization because they kill bacteria.

SPECTRUM & WAVELENGTHS

X-Rays: Wavelengths from 10 nm to .001 nm. These rays have enough energy to penetrate deep into tissues and cause damage to cells; are stopped by dense materials, such as bone.

- Used to look at solid structures, such as bones and bridges (for cracks), and for treatment of cancer.

SPECTRUM & WAVELENGTHS

Gamma Rays: Carry the most energy and have the shortest wavelengths, less than one trillionth of a meter (10^{-12}).

- Gamma rays have enough energy to go through most materials easily; you would need a 3-4 ft thick concrete wall to stop them!

- Gamma rays are released by nuclear reactions in nuclear power plants, by nuclear bombs, and by naturally occurring elements on Earth.
- Sometimes used in the treatment of cancers.

MICROWAVE BAND DESIGNATION

Frequency (GHz)	Wavelength (cm)	IEEE band
1 - 2	30 - 15	L
2 - 4	15 - 7.5	S
4 - 8	7.5 - 3.75	C
8 - 12	3.75 - 2.5	X
12 - 18	2.5 - 1.67	Ku
18 - 27	1.67 - 1.11	K
27 - 40	1.11 - 0.75	Ka
40 - 300	0.75 - 0.1	mm

APPLICATION OF MICROWAVE

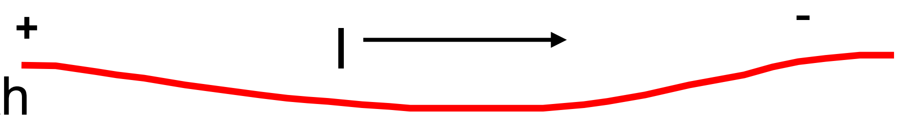
- Communication systems
  - UHF TV
  - Microwave Relay
  - Satellite Communication
  - Mobile Radio
  - Telemetry
- Radar system
  - Search & rescue
  - Airport Traffic Control
  - Navigation
  - Tracking
  - Fire control
  - Velocity Measurement
- Microwave Heating
  - Industrial Heating
  - Home microwave ovens
- Environmental remote sensing
- Medical system
- Test equipment

Week 2

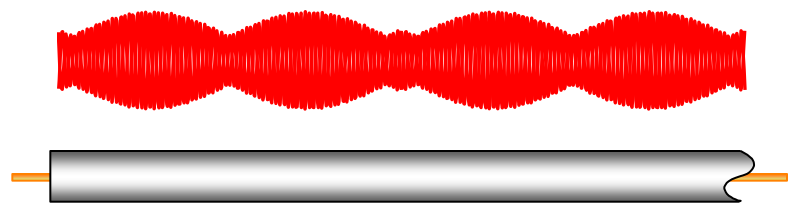
Slide 31-43

TRANSMISSION LINES

Low frequencies



- wavelengths >> wire length
- current (I) travels down wires easily for efficient power transmission
- measured voltage and current not dependent on position along wire

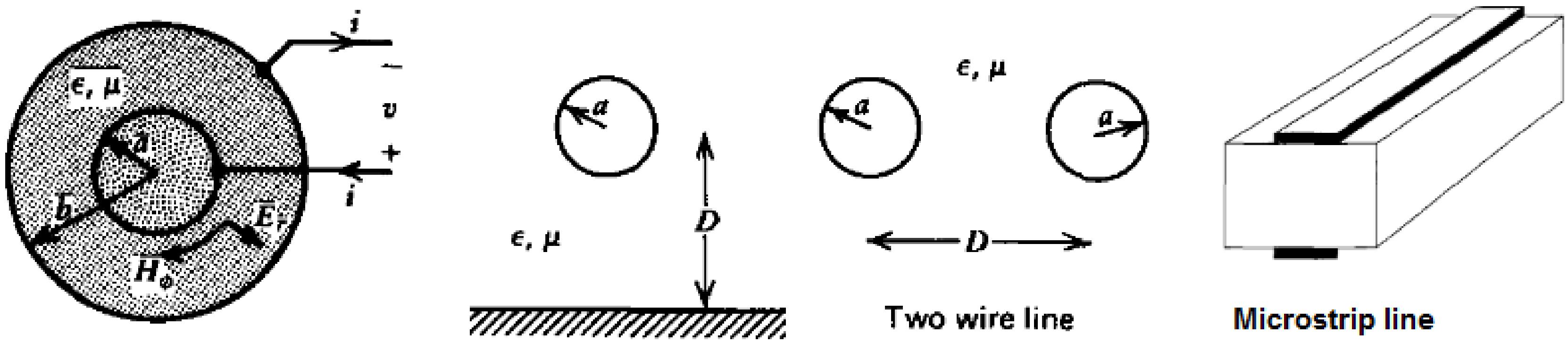


High frequencies

- wavelength << length of transmission medium
- need transmission lines for efficient power transmission
- matching to characteristic impedance (Z_o) is very important for low reflection and maximum power transfer
- measured voltage/current dependent on position along line

Transmission Line Theory

TRANSMISSION LINES



Coaxial cable

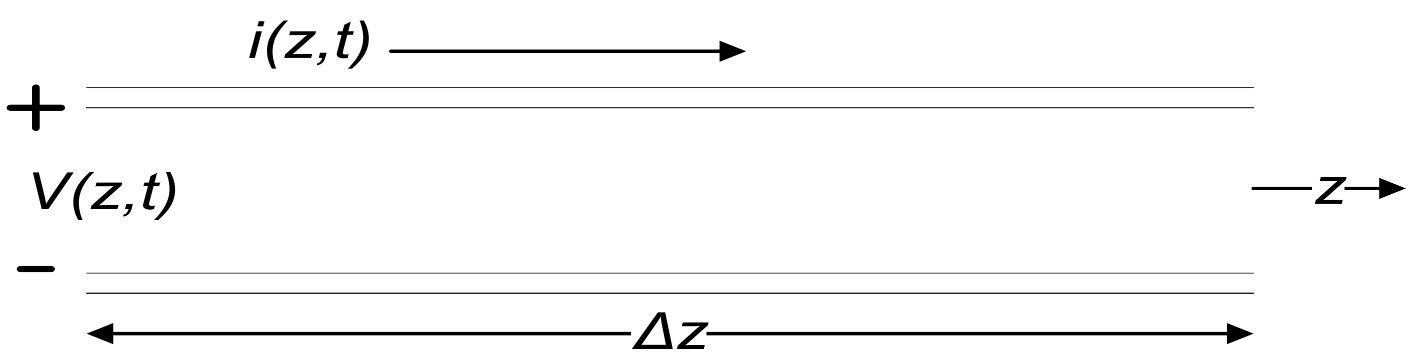
Wire above plane

Two wire line

Microstrip line

TRANSMISSION LINE EQUATIONS

- Transmission line is often schematically represented as a two-wire line.

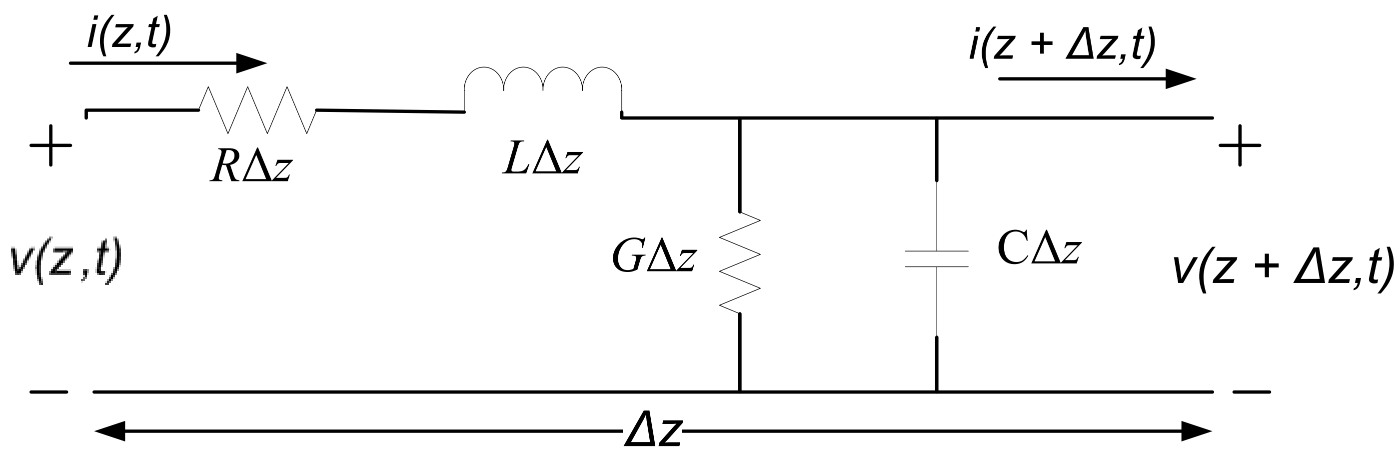


The transmission line always have at least two conductors.

Above figure can be modeled as a lumped-element circuit, as shown, if $\Delta z << \lambda$.

TRANSMISSION LINE EQUATIONS

- The parameters are expressed in their respective name per unit length.

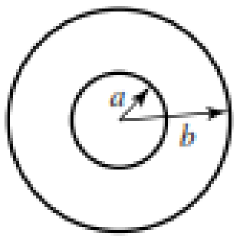
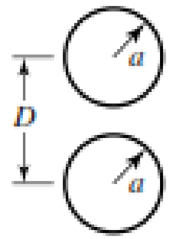
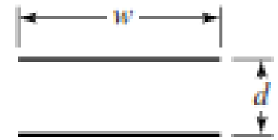


R = series resistant per unit length, for both conductors, in Ω/m
L = series inductance per unit length, for both conductors, in H/m
G = shunt conductance per unit length, in S/m
C = shunt capacitance per unit length, in F/m

TRANSMISSION LINE EQUATIONS

- The series ***L*** represents the total self-inductance of the two conductors.
- The shunt capacitance ***C*** is due to close proximity of the two conductors.
- The series resistance ***R*** represents the resistance due to the finite conductivity of the conductors.
- The shunt conductance ***G*** is due to dielectric loss in the material between the conductors.
- NOTE: ***R*** and ***G***, represent loss.

TRANSMISSION LINE PARAMETERS

	COAX	TWO-WIRE	PARALLEL PLATE
			
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

TRANSMISSION LINE EQUATIONS (1.3)

- By using the Kirchoff's voltage law around the loop over the line of length Δz, and Kirchoff's current law at the capacitive node we have:

$$-\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} \qquad [1]$$

$$-\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C\frac{\partial v(z,t)}{\partial t} \qquad [2]$$

These equations are known as Telegrapher equations.

TRANSMISSION LINE EQUATIONS

- From the above equations, the wave equation for *V*(*z*) and *I*(*z*) in frequency domain can be written as:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \quad [3] \qquad \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \quad [4]$$

where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad [5]$$

γ is the complex propagation constant, which is function of frequency.
α is the attenuation constant in nepers per unit length,
β is the phase constant in radians per unit length.

PROPAGATION CONSTANT

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\alpha = \sqrt{\frac{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} + (RG - \omega^2 LC)}{2}}$$

$$\beta = \sqrt{\frac{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - (RG - \omega^2 LC)}{2}}$$

TRANSMISSION LINE EQUATIONS

- The traveling wave solution to the equation [4] and [5] can be found as:

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z} \qquad [6]$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z} \qquad [7]$$

- I(z) can also be obtained as:

$$I(z) = \frac{\gamma}{R + j\omega L}\left(V_+ e^{-\gamma z} - V_- e^{\gamma z}\right)$$

TRANSMISSION LINE EQUATIONS

The characteristic impedance, Z_0 can be defined as:

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad [8]$$

Note: characteristic impedance (Z_o) is the ratio of voltage to current in a forward travelling wave. Z_0 is usually a real impedance (e.g. 50 or 75 ohms).

TRANSMISSION LINE EQUATIONS

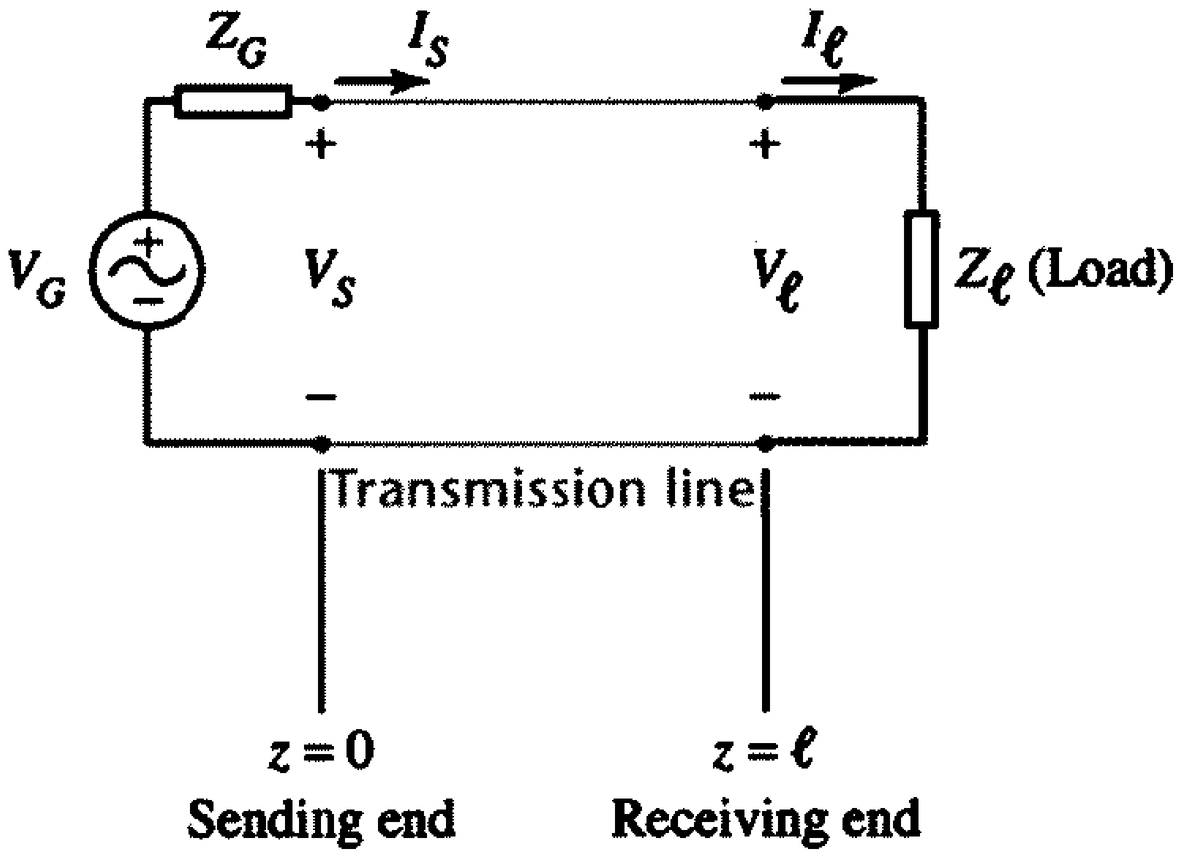
$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I(z) = Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z})$$

Week 3

Slide 45-64

SOLUTION IN TERMS OF LOAD QUANTITIES



SOLUTION IN TERMS OF LOAD QUANTITIES

$$V(z) = (1/2)[V_l(e^{-\gamma(z-l)} + e^{\gamma(z-l)}) + I_l Z_0(e^{-\gamma(z-l)} - e^{\gamma(z-l)})]$$

$$I(z) = (Y_0/2)[V_l(e^{-\gamma(z-l)} - e^{\gamma(z-l)}) + I_l Z_0(e^{-\gamma(z-l)} + e^{\gamma(z-l)})]$$

$$V_s = V(z=0) = V_l \cos\beta l + jI_l Z_0 \sin\beta l$$

$$I_s = I(z=0) = jY_0 V_l \sin\beta l + I_l \cos\beta l$$

SOLUTION IN TERMS OF SOURCE QUANTITIES

$$V(z) = (1/2)[V_s(e^{-\gamma z} + e^{\gamma z}) + I_s Z_0(e^{-\gamma z} - e^{\gamma z})]$$

$$I(z) = (Y_0/2)[V_s(e^{-\gamma z} - e^{\gamma z}) + I_s Z_0(e^{-\gamma z} + e^{\gamma z})]$$

$$V_l = V(z = l) = V_s \cos \beta l - jI_s Z_0 \sin \beta l$$

$$I_l = I(z = l) = -jY_0 V_s \sin \beta l + I_s \cos \beta l$$

Propagation constant, Characteristic Impedance and Phase velocity for various types of Transmission Line

TRANSMISSION LINE EQUATIONS

- Lossless Line:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$$

$$\therefore \alpha = 0 \text{ and } \beta = \omega\sqrt{LC}$$

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

TRANSMISSION LINE EQUATIONS

- Low Loss Line (R<<jωL and G<<jωC):

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\approx j\omega\sqrt{LC}\left(1 + \frac{R}{j2\omega L} + \frac{G}{j2\omega C}\right)$$

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\text{and } \beta = \omega\sqrt{LC}$$

$$Z_0 \approx \sqrt{\frac{L}{C}}\left(1 - \frac{j}{2\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\right)$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

TRANSMISSION LINE EQUATIONS

- Distortion-less Line (R/L=G/C):

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(R + j\omega L)(\frac{RC}{L} + j\omega C)}$$

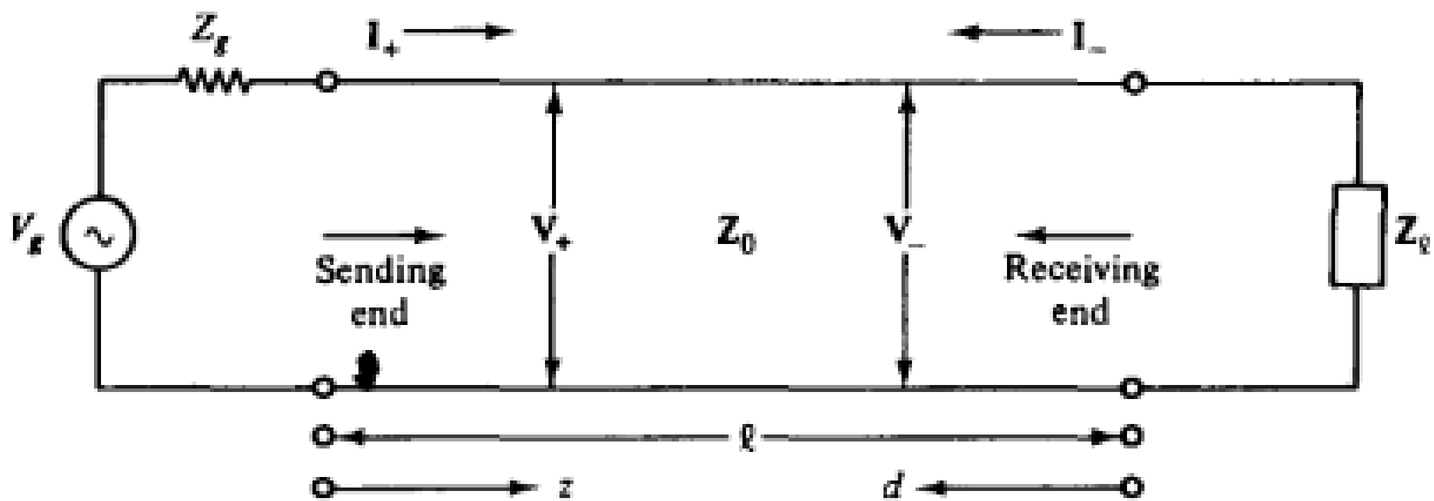
$$\alpha = R\sqrt{\frac{C}{L}} \quad \text{and } \beta = \omega\sqrt{LC}$$

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R + j\omega L}{(RC/L) + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Reflection and Transmission Coefficients

REFLECTION & TRANSMISSION



The ratio of the reflected and the incident waves at any position, is defined as the reflection coefficient at that point and is given by:

$$\Gamma_z = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}}. \quad \Rightarrow \quad \Gamma_l = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_l - Z_0}{Z_l + Z_0}$$

REFLECTION & TRANSMISSION

The reflection coefficient at a distance d from the load towards the generator is

$$\Gamma_{z=l-d} = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}} = \Gamma_l e^{-2\gamma d}$$

The Transmission coefficient T is given by:

$$T = \frac{V_{tr} e^{-\gamma l}}{V_+ e^{-\gamma l}} = \frac{2Z_l}{Z_l + Z_0}$$

$$\boxed{(1+\Gamma_l) = T}$$

Prepared By- Noor Md Shahriar, Senior Lecturer, Dept. of EEE, UGV

REFLECTION & TRANSMISSION

The incident power to the load and reflected power from the load are

$$P_{in} = \frac{1}{2}\mathrm{Re}\,al\left(V_{in}I_{in}^{*}\right) = \frac{1}{2}\mathrm{Re}\,al\left(Z_0 I_{in} I_{in}^{*}\right) = \frac{\left(\left|V_+\right|e^{-\alpha l}\right)^2}{2Z_0}$$

$$P_{ref} = \frac{1}{2}\mathrm{Re}\,al\left(V_{ref}I_{ref}^{*}\right) = \frac{1}{2}\mathrm{Re}\,al\left(Z_0 I_{ref} I_{ref}^{*}\right) = \frac{\left(\left|V_-\right|e^{\alpha l}\right)^2}{2Z_0}$$

REFLECTION & TRANSMISSION

The power delivered to the load is

$$P_l = \frac{\left(|V_+|e^{-\alpha l}\right)^2}{2Z_0} - \frac{\left(|V_-|e^{\alpha l}\right)^2}{2Z_0}$$

$$= \frac{\left(|V_+|e^{-\alpha l}\right)^2}{2Z_0} - \frac{\left(|V_+\Gamma_l|e^{-\alpha l}\right)^2}{2Z_0} = \frac{\left(|V_+|e^{-\alpha l}\right)^2}{2Z_0}\left(1 - |\Gamma_l|^2\right)$$

If $Z_0 \neq Z_l$, not all power goes to load. This loss in power is known as Return Loss (RL) which is given by:

RL= -20 log$|\Gamma_l|$ dB.

REFLECTION & TRANSMISSION

The power delivered to the load is the transmitted power

$$\frac{\left(\left|V_+\right|e^{-\alpha l}\right)^2}{2Z_0}\left(1-\left|\Gamma_l\right|^2\right)=\frac{\left(\left|V_{tr}\right|e^{-\alpha l}\right)^2}{2Z_l}$$

$$\Rightarrow 1-\left|\Gamma_l\right|^2=\frac{Z_0}{Z_l}T^2$$

The transmission coefficient is often expressed in dB as the Insertion Loss (IL) which is given by:

IL= -20 log $\left|T\right|$ dB.

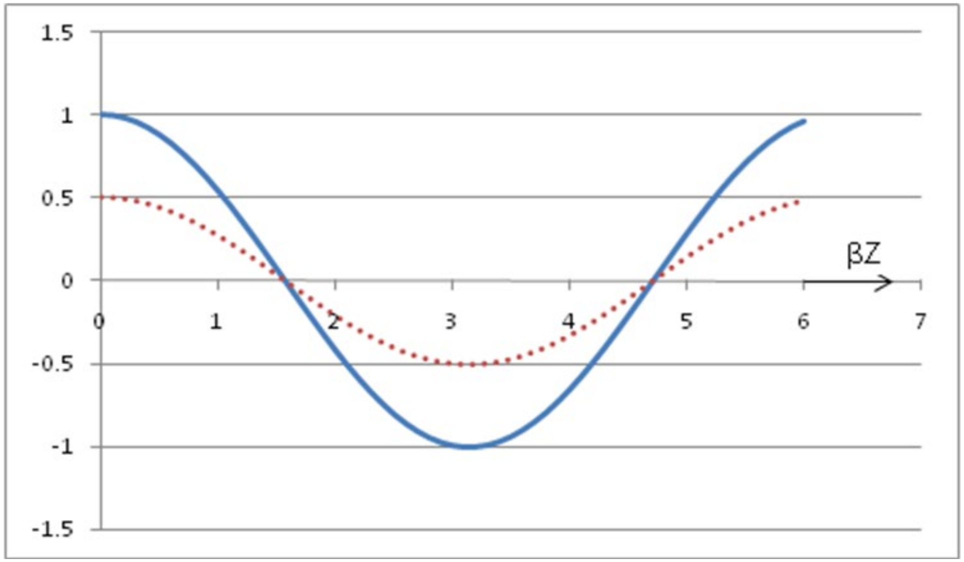
Standing wave
and
Standing wave ratio

STANDING WAVE

- Pure Standing Wave ($\left|V_+ e^{-\alpha z}\right| = \left|V_- e^{\alpha z}\right|$)

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z} = 2V_+ e^{-\alpha z} \cos \beta z$$

$$v(z,t) = \mathrm{Re}\,al\left[V(z) e^{j\omega t}\right] = 2V_+ e^{-\alpha z} \cos \beta z \cos \omega t$$

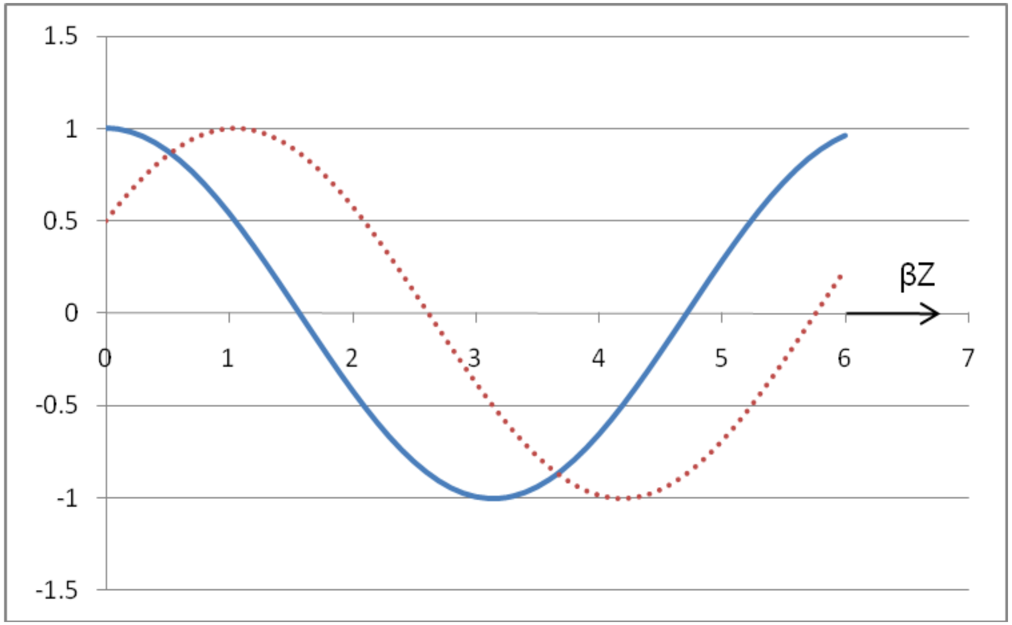


TRAVELING WAVE

- Pure Traveling Wave ($V_- e^{\gamma z} = 0$)

$$V(z) = V_+ e^{-\gamma z} = V_+ e^{-(\alpha + j\beta)z}$$

$$v(z,t) = V_+ e^{-\alpha z} \cos(\omega t - \beta z)$$



STANDING WAVE RATIO

- Standing Wave Ratio

$$V(z=l-d)=V_+e^{-\gamma z}+V_-e^{\gamma z}=V_+e^{-\gamma(l-d)}\left(1+\Gamma_l e^{-2\gamma d}\right)$$

$$=V_+e^{-\gamma(l-d)}\left[1+\left|\Gamma_l\right|e^{-2\alpha d}\left(\cos\phi+j\sin\phi\right)\right]$$

$$\Rightarrow\left|V(z)\right|=\left|V_+\right|e^{-\alpha(l-d)}\left[1+a^2+2a\cos\phi\right]^{1/2}$$

$$\text{where a}=\left|\Gamma_l\right|\text{e}^{-2\alpha\text{d}}\ \text{and}\ \ \phi=\theta_l-2\beta d$$

STANDING WAVE RATIO

$$|V(z)| = |V_+| e^{-\alpha(l-d)} \left[1 + a^2 + 2a\cos\phi\right]^{1/2}$$

$$V_{\max} = |V_+| e^{-\alpha(l-d)} \left[1 + |\Gamma_{z=l-d}|\right]$$

When $\phi = \theta_l - 2\beta d = -2n\pi$, where n=0, 1, 2,………

$$V_{\min} = |V_+| e^{-\alpha(l-d)} \left[1 - |\Gamma_{z=l-d}|\right]$$

When $\phi = \theta_l - 2\beta d = -(2n+1)\pi$, where n=0, 1, 2,………

STANDING WAVE RATIO

$$\rho \equiv \frac{|V_{\max}|}{|V_{\min}|} = \frac{|I_{\max}|}{|I_{\min}|} = \frac{1+|\Gamma_{z=l-d}|}{1-|\Gamma_{z=l-d}|}$$

$$= \frac{1+|\Gamma_l|e^{-2\alpha d}}{1-|\Gamma_l|e^{-2\alpha d}} = \frac{1+|\Gamma_l|}{1-|\Gamma_l|}$$

$$\boxed{|\Gamma_l| = \frac{\rho - 1}{\rho + 1}}$$

Standing Wave Ratio is not defined for lossy line.

- SWR (ρ) is a real number such that $1 \leq \rho \leq \infty$
- $\rho=1$ implies a matched load

ASSIGNMENT-1

The VSWR on a 50-Ohm (characteristic impedance) transmission line is 2. The distance between successive voltage minima is 40 cm while the distance from the load to the first minima is 10 cm. What is the reflection coefficient and load impedance?

The magnitude and phase angle of reflection coefficient can be obtained from:

$$\left|\Gamma_l\right| = \frac{\rho - 1}{\rho + 1} \qquad \phi = \theta_l - 2\beta d = -(2n+1)\pi$$

Load impedance can be obtained from $\Gamma_l = \dfrac{Z_l - Z_0}{Z_l + Z_0}$

Week 4

Slide 66-82

Line Impedance

TRANSMISSION LINES

- Lossy Line

$$Z \equiv \frac{V(d)}{I(d)} = Z_0 \frac{V_+ e^{-\gamma(l-d)} + V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)} - V_- e^{\gamma(l-d)}}$$

$$= Z_0 \frac{Z_l \cosh(\gamma d) + Z_0 \sinh(\gamma d)}{Z_l \sinh(\gamma d) + Z_0 \cosh(\gamma d)}$$

$$\boxed{Z = Z_0 \frac{1+\Gamma_d}{1-\Gamma_d}}$$

$$\Gamma_d = \Gamma_l e^{-2\gamma d} = |\Gamma_l| e^{-2\alpha d} e^{j(\theta_l - 2\beta d)}$$

- Lossless Line

$$Z = Z_0 \frac{Z_l \cos(\beta d) + jZ_0 \sin(\beta d)}{Z_0 \cos(\beta d) + jZ_l \sin(\beta d)}$$

$$\boxed{Z = Z_0 \frac{1+\Gamma_d}{1-\Gamma_d}}$$

$$\Gamma_d = \Gamma_l e^{-2\gamma d} = |\Gamma_l| e^{j(\theta_l - 2\beta d)}$$

Line impedance repeats itself every λ/2 distance along a lossless transmission line and inverts itself every λ/4 distance along the lossless transmission line.

TRANSMISSION LINES

- Significant features of line impedance:

Line impedance varies along the line and

1) $$\bar{z}_{\max} = \frac{Z_{\max}}{Z_0} = \frac{|V_{\max}|}{Z_0 |I_{\min}|} = \frac{1+|\Gamma_l|}{1-|\Gamma_l|} = \rho$$

2) $$\bar{z}_{\min} = \frac{Z_{\min}}{Z_0} = \frac{|V_{\min}|}{Z_0 |I_{\max}|} = \frac{1-|\Gamma_l|}{1+|\Gamma_l|} = \frac{1}{\rho}$$

3) $$\bar{z}_{\max}\left(d \pm \frac{\lambda}{4}\right) = \frac{1}{\bar{z}_{\min}(d)}$$

ASSIGNMENT-2

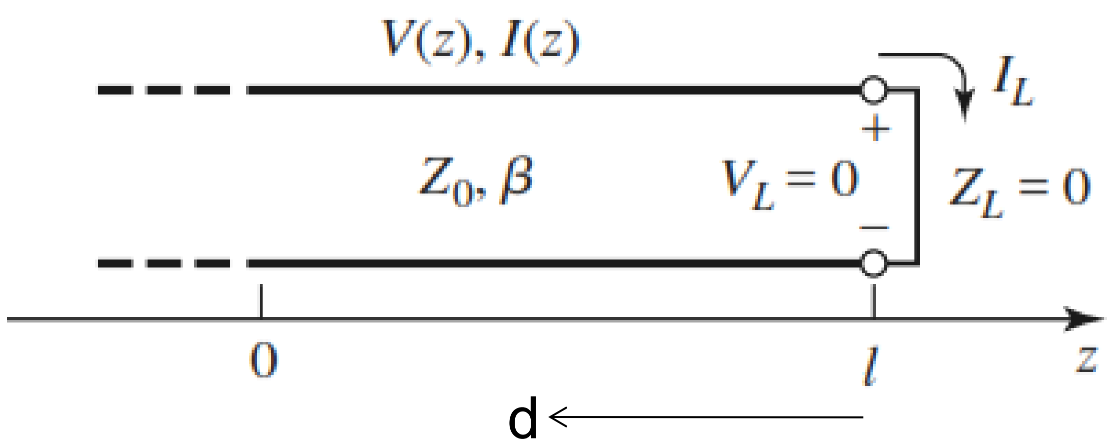
A generator having an internal resistance of 50 Ω applies a signal with a wavelength of 100 cm to a lossless transmission line. The transmission line is 50 cm long and terminated with a load having an impedance of $50 + j20\ \Omega$. The peak value of the open-circuit terminal voltage at the source is 10 V. The per-unit-length inductance and capacitance of the line are $0.17\ \mu H/m$ and 70 pF/m, respectively. Calculate (a) the frequency of the source and (b) the voltage, current, and power at the sending and receiving ends of the transmission line.

ASSIGNMENT-3

A load impedance of $40 + j70\ \Omega$ terminates a $100\ \Omega$ transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the standing wave ratio on the line, and the return loss.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

- Short circuited transmission line (Z_L=0)
- The reflection coefficient Γ_l = -1.
- Then, the standing wave ratio (ρ) is infinite.



SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

- The voltage and current on the line are:

$$V(d) = V_+ e^{-j\beta(l-d)}\left(1 - e^{-j2\beta d}\right) = V_+ e^{-j\beta l}\left(e^{j\beta d} - e^{-j\beta d}\right) = j2V_+ e^{-j\beta l} \sin\beta d$$

$$v(d,t) = \mathrm{Re}\,al\left[j2V_+ \sin\beta d e^{j(\omega t - \beta l)}\right] = -2V_+ \sin\beta d \sin(\omega t - \beta l)$$

$$i(d,t) = \mathrm{Re}\,al\left[2Y_0 V_+ \cos\beta d e^{j(\omega t - \beta l)}\right] = 2Y_0 V_+ \cos\beta d \cos(\omega t - \beta l)$$

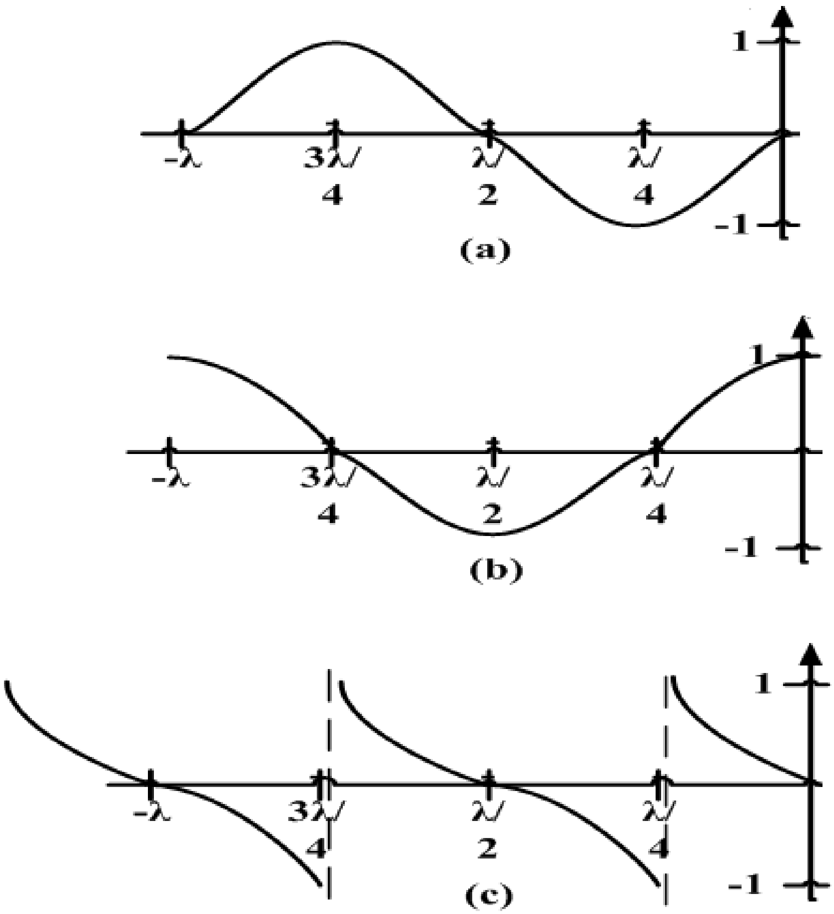
The input impedance to this line is:

$$Z_{in} = jZ_0 \tan\beta d$$

When d = 0 we have Z_{in}=0, but for d = λ/4 we have $Z_{in} = \infty$ (open circuit)

Moreover, the impedance is periodic in *d*.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES



Nor. V (peak value)

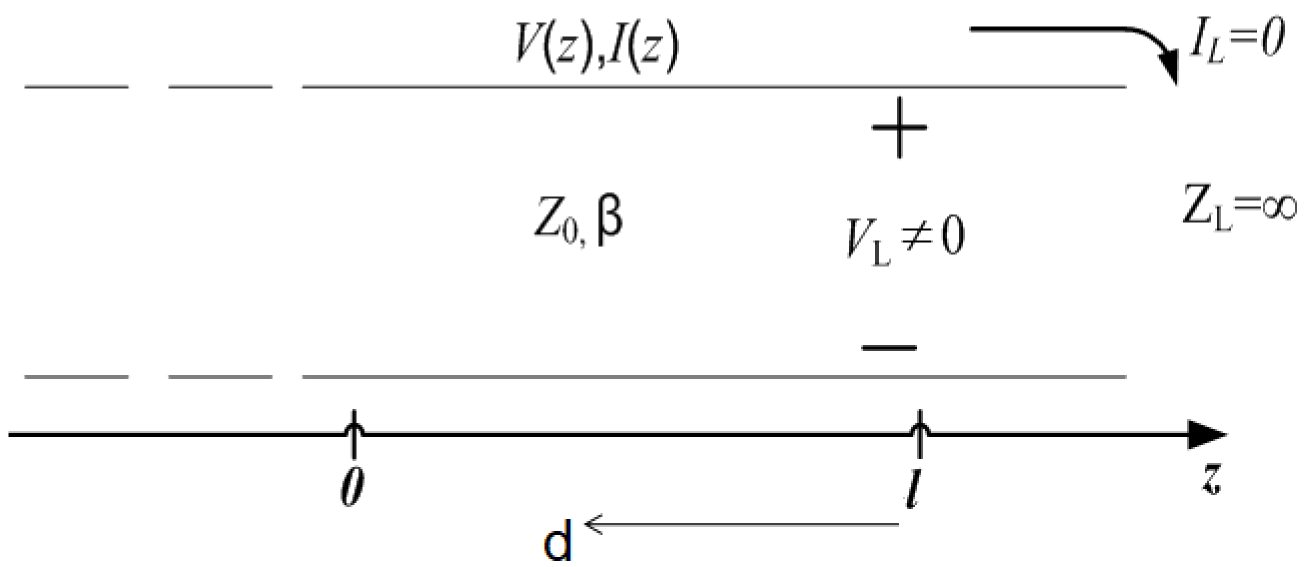
Nor. I (peak value)

Nor. Z

(a) Voltage (b) Current (c) impedance variation along a short circuited transmission line

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

- Open circuit Transmission Line (Z_L=∞)
- The reflection coefficient is Γ=1.
- The standing wave ratio is infinite.



A transmission line terminated in an open circuit.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

- The voltage and current on the line are:

$$V(d) = V_+ e^{-j\beta(l-d)}\left(1 + e^{-j2\beta d}\right) = V_+ e^{-j\beta l}\left(e^{j\beta d} + e^{-j\beta d}\right) = 2V_+ e^{-j\beta l} \cos \beta d$$

$$v(d,t) = \mathrm{Re}\,al\left[2V_+ \cos \beta d e^{j(\omega t - \beta l)}\right] = 2V_+ \cos \beta d \cos(\omega t - \beta l)$$

$$i(d,t) = \mathrm{Re}\,al\left[j2Y_0 V_+ \sin \beta d e^{j(\omega t - \beta l)}\right] = -2Y_0 V_+ \sin \beta d \sin(\omega t - \beta l)$$

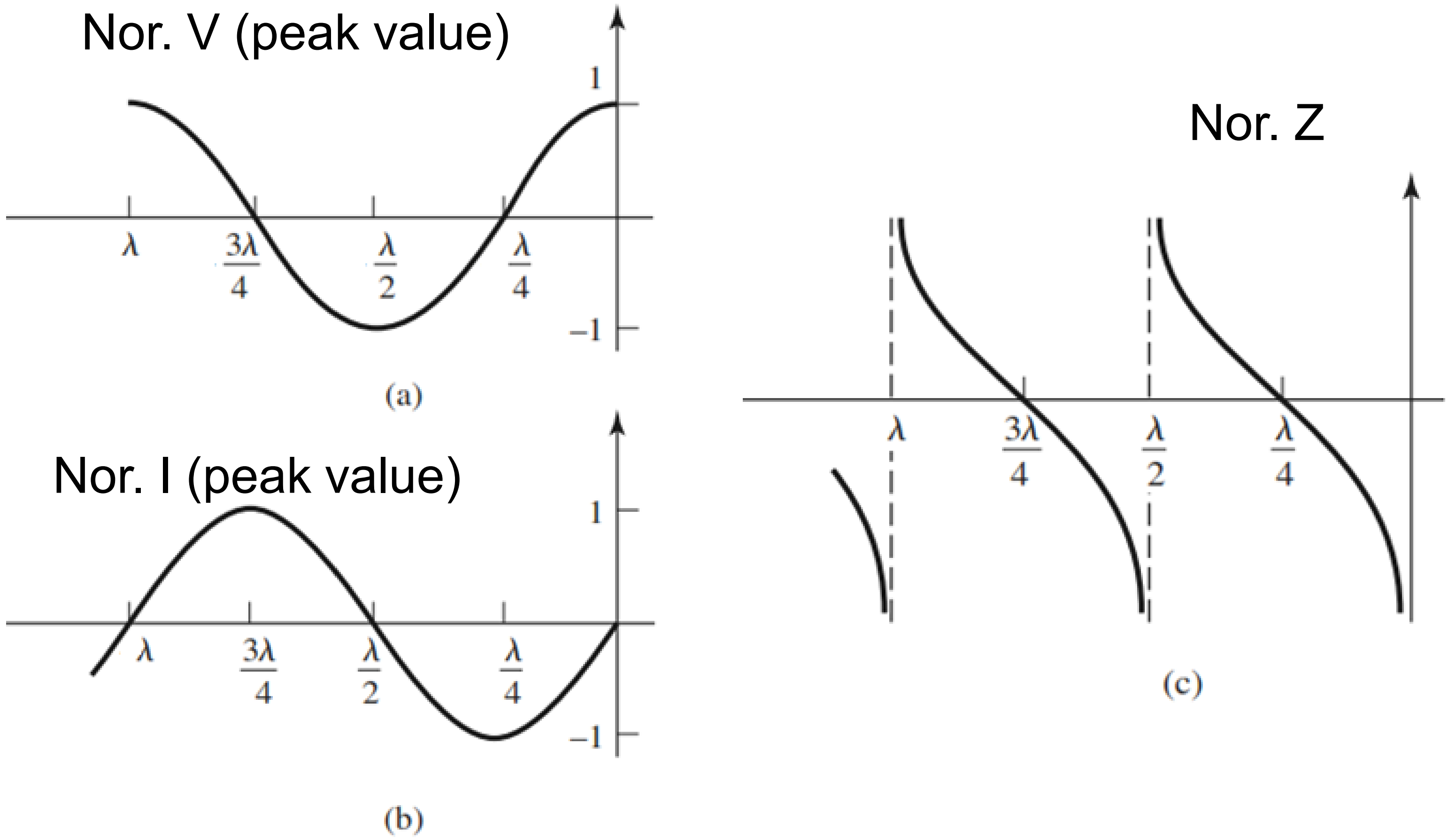
The input impedance to this line is:

$$Z = -jZ_0 \cot(\beta d)$$

When d = 0 we have $Z_{in} = \infty$ (open circuit), but for d = $\lambda/4$ we have $Z_{in} = 0$.

Moreover, the impedance is periodic in *d*.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES



(a) Voltage (b) Current (c) impedance variation along an open circuit transmission line.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

When the transmission line are terminated with some special lengths such as $l = \lambda/2$,

$$Z_{in} = Z_L$$

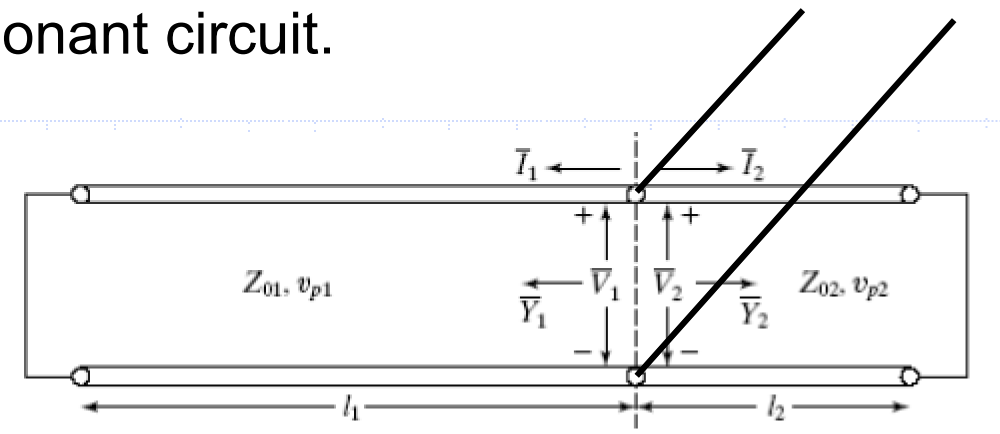
For $l = \lambda/4 + n\lambda/2$, and n =0, 1, 2, 3, … The input impedance is given by:

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Note: This is also known as *quarter wave transformer*.

RESONANT CIRCUIT AT MICROWAVE FREQUENCY

Since a section of a short/open circuited line acts like inductance and capacitance, they may connect in series or parallel to form a series/parallel circuit. In the following, two short circuited lines are connected in parallel to form a parallel resonant circuit.



RESONANT CIRCUIT AT MICROWAVE FREQUENCY

$$\overline{V}_1 = \overline{V}_2 \qquad \overline{I}_1 + \overline{I}_2 = 0 \longrightarrow \frac{\overline{I}_1}{\overline{V}_1} + \frac{\overline{I}_2}{\overline{V}_2} = 0 \longrightarrow \overline{Y}_1 + \overline{Y}_2 = 0$$

$$\overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{jZ_{01}\tan\beta_1 l_1} = \frac{1}{jZ_{01}\tan(2\pi f/v_{p1})l_1}$$

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{jZ_{02}\tan\beta_2 l_2} = \frac{1}{jZ_{02}\tan(2\pi f/v_{p2})l_2}$$

Characteristic equation for resonant frequency:

$$Z_{01}\tan\frac{2\pi f}{v_{p1}}l_1 + Z_{02}\tan\frac{2\pi f}{v_{p2}}l_2 = 0$$

ASSIGNMENT-4

A quarter-wavelength transmission line with a characteristic impedance of 400 Ω is short-circuited at the receiving end. Across the sending-end terminals of the line there is a generator with an open-circuit voltage of $v(t) = 50 \cos 10^6 t$ V and an internal resistance of 100 Ω. Determine the voltage across the receiving end and at the midpoint of the transmission line.

SUMMARY

- Three parameters to measure the 'goodness' or 'perfectness' of the termination of a transmission line are:
  1. Reflection coefficient (Γ_l)
  2. Standing Wave Ratio (SWR)
  3. Return loss (RL)

Week 5
Slide 84-98

EXAMPLE

Calculate the SWR, reflection coefficient magnitude, $|\Gamma_l|$ and return loss values to complete the entries in the following table:

SWR	$\|\Gamma_l\|$	RL (dB)
1.00	0.00	∞
1.01		
	0.01	
		30.0
2.50		

EXAMPLE

The formulas that should be used in this calculation are as follow:

$$RL = -20\log\left|\Gamma_l\right|$$

$$SWR = \frac{1+\left|\Gamma_l\right|}{1-\left|\Gamma_l\right|}$$

$$\left|\Gamma_l\right| = 10^{-(RL/20)}$$

$$\left|\Gamma_l\right| = \frac{SWR-1}{SWR+1}$$

EXAMPLE

Calculate the SWR, reflection coefficient magnitude, $|\Gamma_l|$ and return loss values to complete the entries in the following table:

SWR	$\|\Gamma_l\|$	RL (dB)
1.00	0.00	∞
1.01	0.005	46.0
1.02	0.01	40.0
1.07	0.0316	30.0
2.50	0.429	7.4

EXAMPLE

A source with 50 Ω source impedance drives a 50 Ω transmission line that is 1/8 of wavelength long, terminated in a load Z_L = 50 – j25 Ω. Calculate:

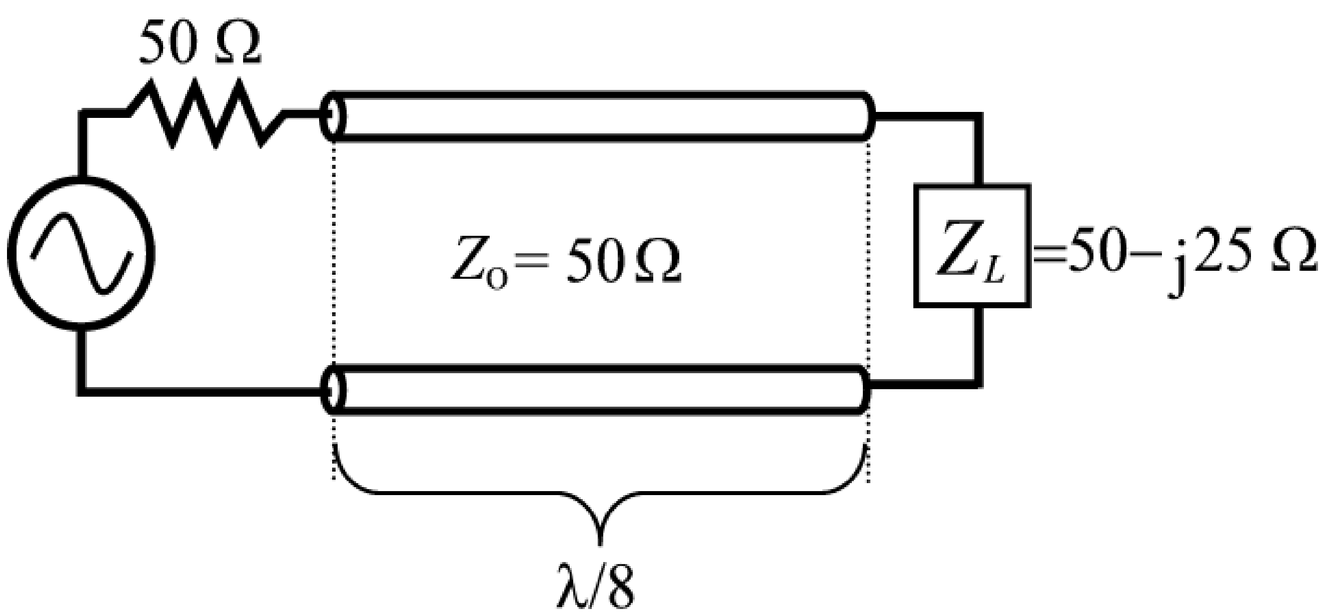
(i) The reflection coefficient, Γ_L

(ii) VSWR

(iii) The input impedance seen by the source.

SOLUTION TO EXAMPLE

It can be shown as:



SOLUTION TO EXAMPLE

(i) The reflection coefficient,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{(50 - j25) - 50}{(50 - j25) + 50} = 0.242e^{-j76^0}$$

(ii) VSWR

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.64$$

SOLUTION TO EXAMPLE

(iii) The input impedance seen by the source, Z_{in}

Need to calculate $\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$ $\therefore \tan\frac{\pi}{4} = 1$

Therefore,

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

$$= 50\frac{50 - j25 + j50}{50 + j50 + 25}$$

$$= 30.8 - j3.8\Omega$$

Smith Chart

SMITH CHART

Smith chart is a graphical plot of the normalized resistance and reactance in the complex plane of reflection coefficient Γ. It is very convenient for transmission line analyses and also a useful tool in impedance matching circuit design.

$$Z_L = Z_0 \frac{1+\Gamma_l}{1-\Gamma_l}$$

$$\Rightarrow \bar{z}_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma_l}{1-\Gamma_l} = \bar{r} + j\bar{x}$$

SMITH CHART

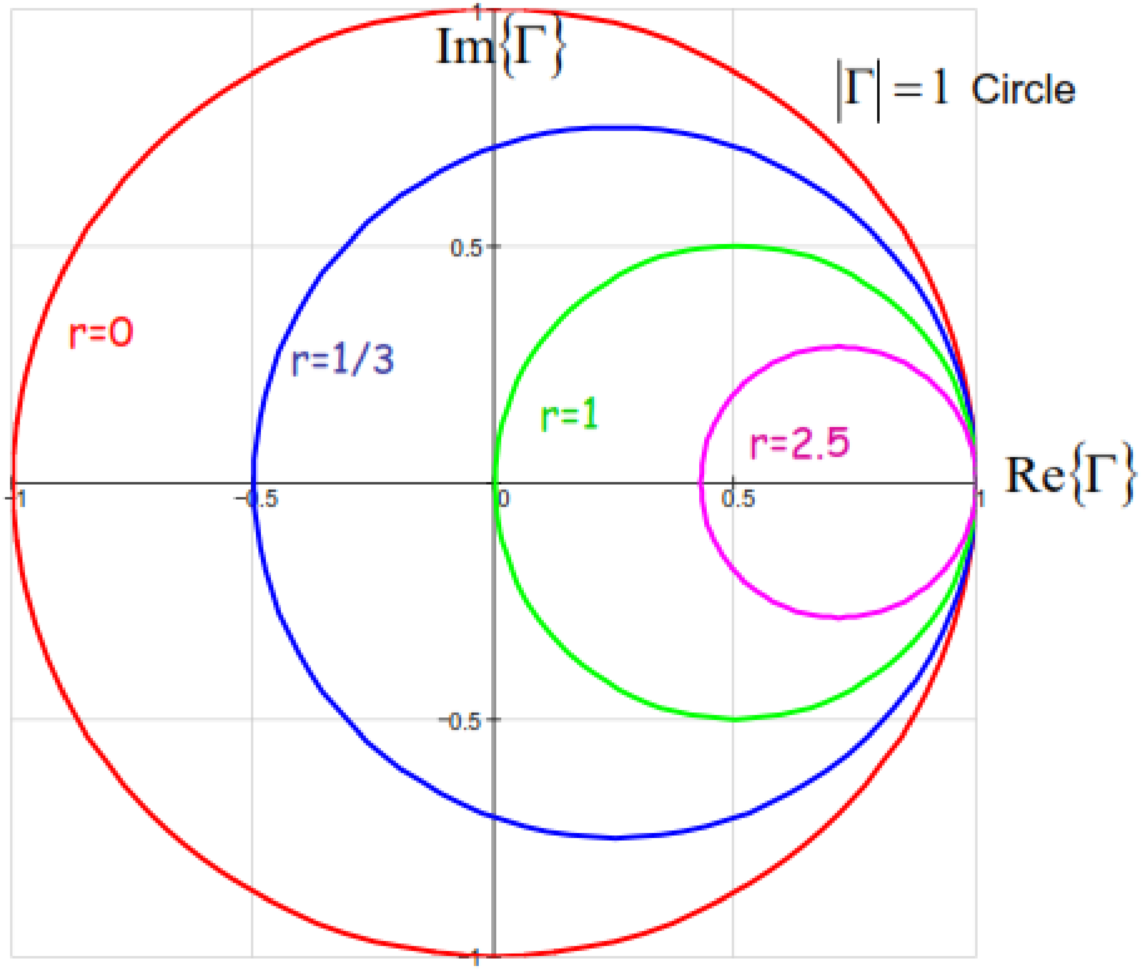
$$\bar{r} + j\bar{x} = \frac{1+\Gamma_l}{1-\Gamma_l}$$

$$\therefore \bar{r} = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$ and $$\bar{x} = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

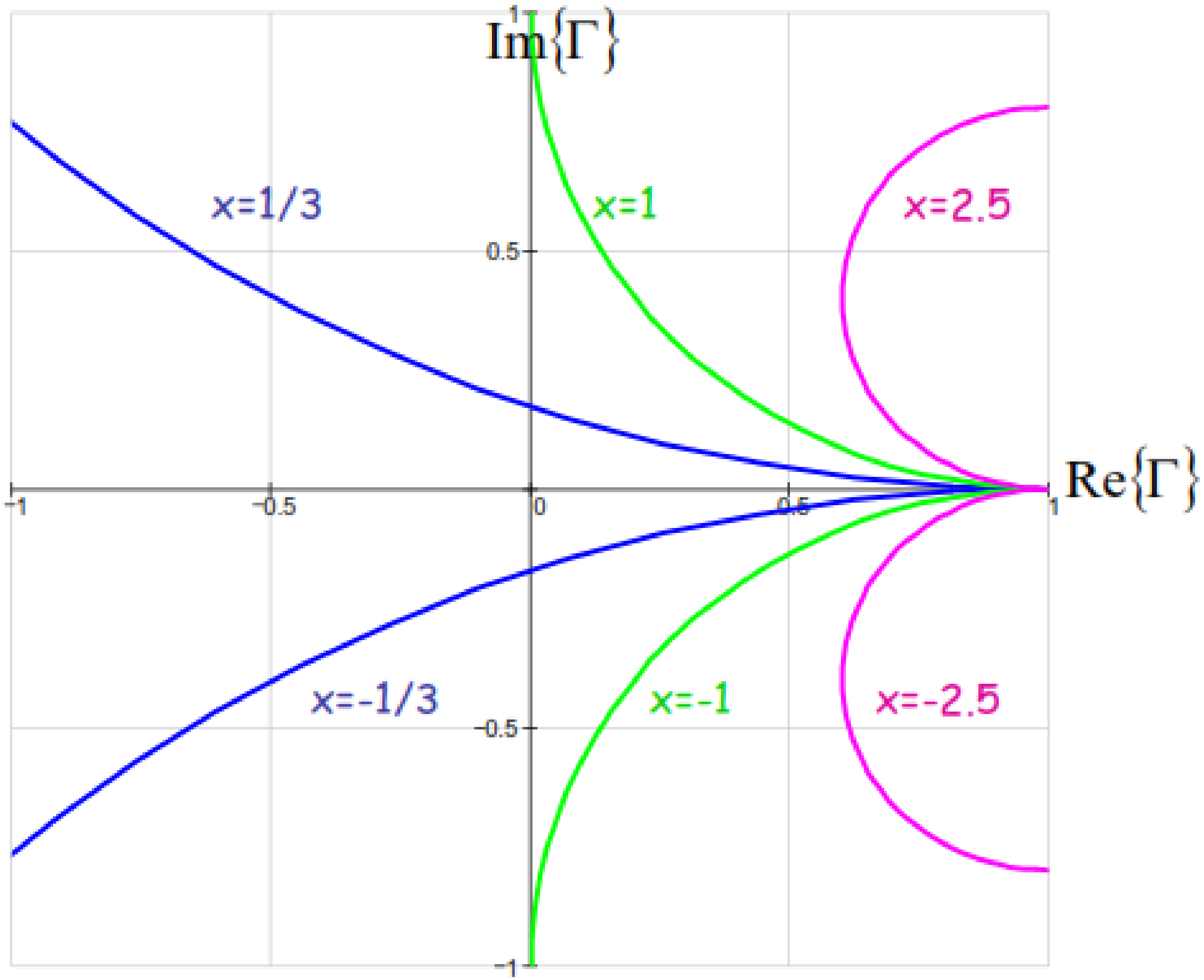
$$\left(\Gamma_r - \frac{\bar{r}}{1+\bar{r}}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+\bar{r}}\right)^2 \qquad (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{\bar{x}}\right)^2 = \left(\frac{1}{\bar{x}}\right)^2$$

The last two equations of r and x define two families of circles in the complex plane of Γ.

SMITH CHART



SMITH CHART



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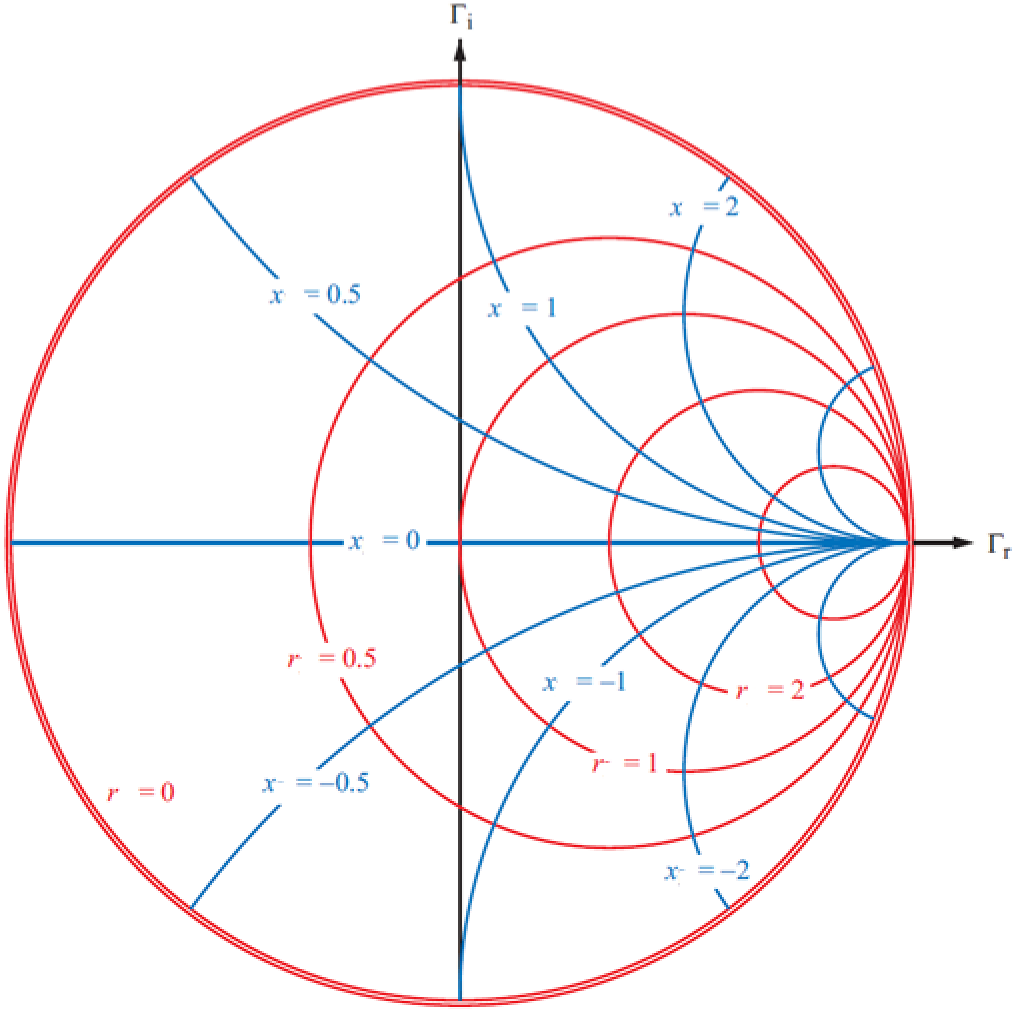
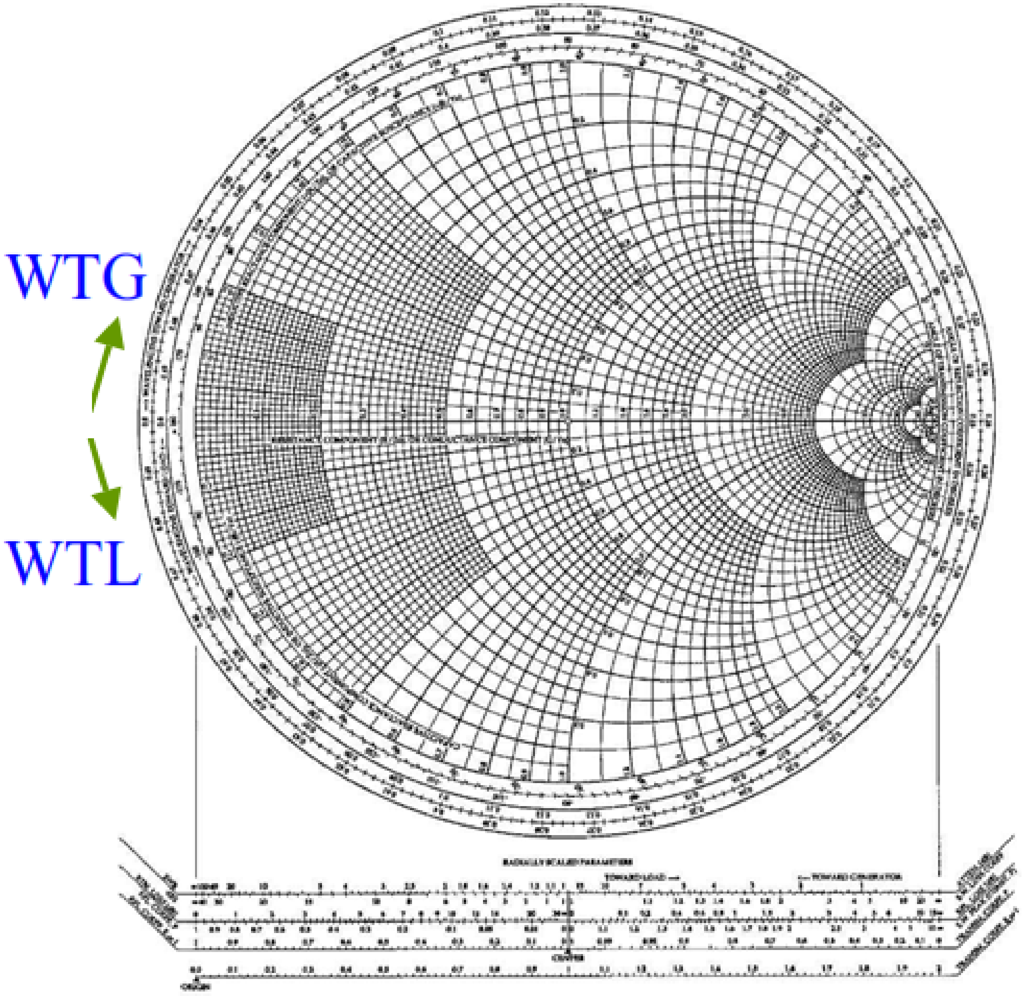
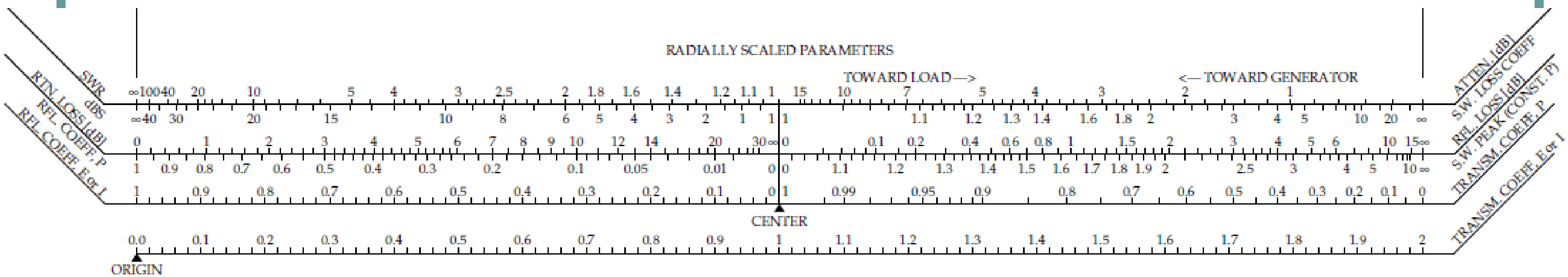


SMITH CHART



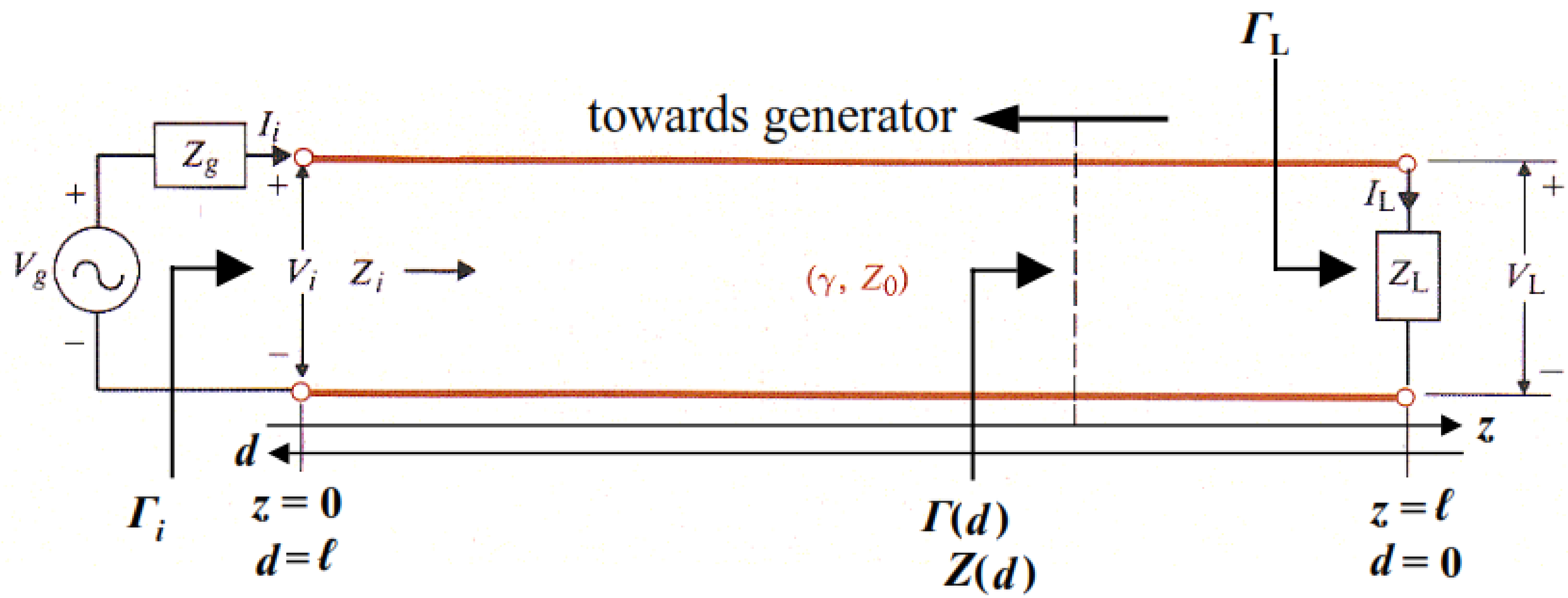

RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN

Week 6

Slide 100-126

SMITH CHART

Smith chart and transmission lines



Reflection coefficient at the load

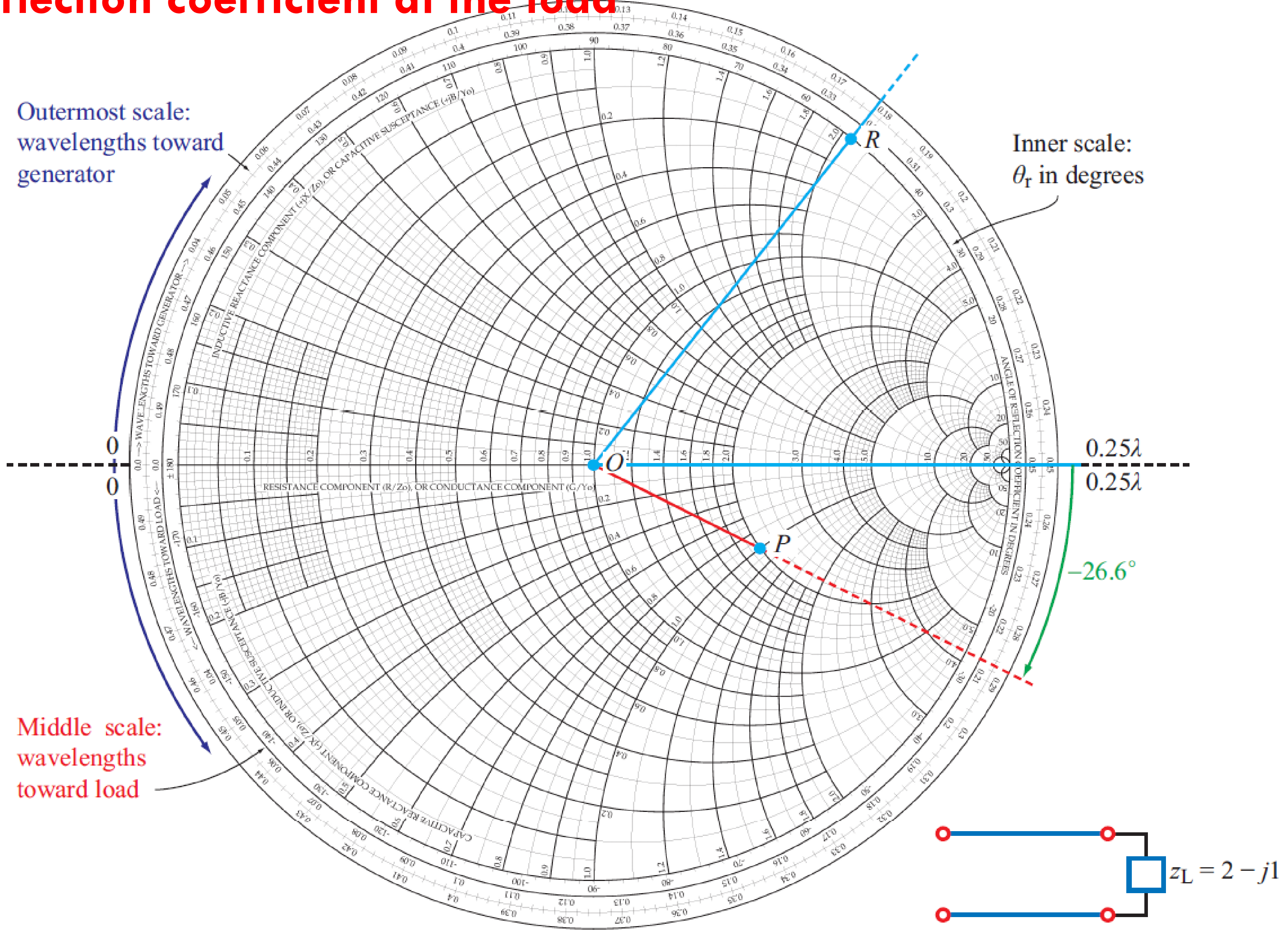


Figure 2-26: Point P represents a normalized load impedance $z_L = 2 - j1$. The reflection coefficient has a magnitude $|\Gamma| = \overline{OP}/\overline{OR} = 0.45$ and an angle $\theta_r = -26.6°$. Point R is an arbitrary point on the $r_L = 0$ circle (which also is the $|\Gamma| = 1$ circle).

Input Impedance

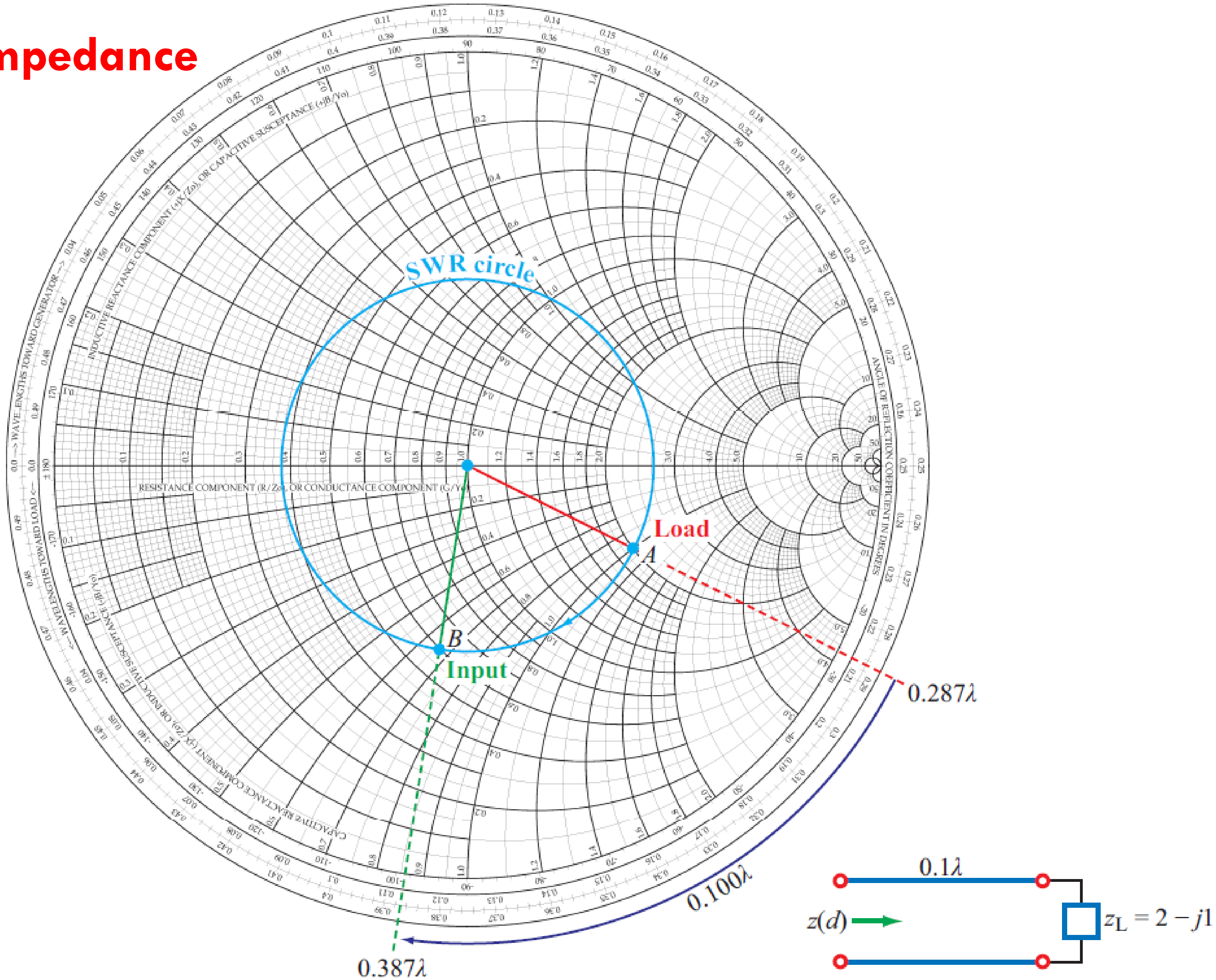


Figure 2-27: Point A represents a normalized load $z_L = 2 - j1$ at 0.287λ on the WTG scale. Point B represents the line input at $d = 0.1\lambda$ from the load. At B, $z(d) = 0.6 - j0.66$.

Maxima and Minima

S is numerically equal to the value of r_0 at P_{max}, the point at which the SWR circle intersects the real Γ axis to the right of the chart's center.

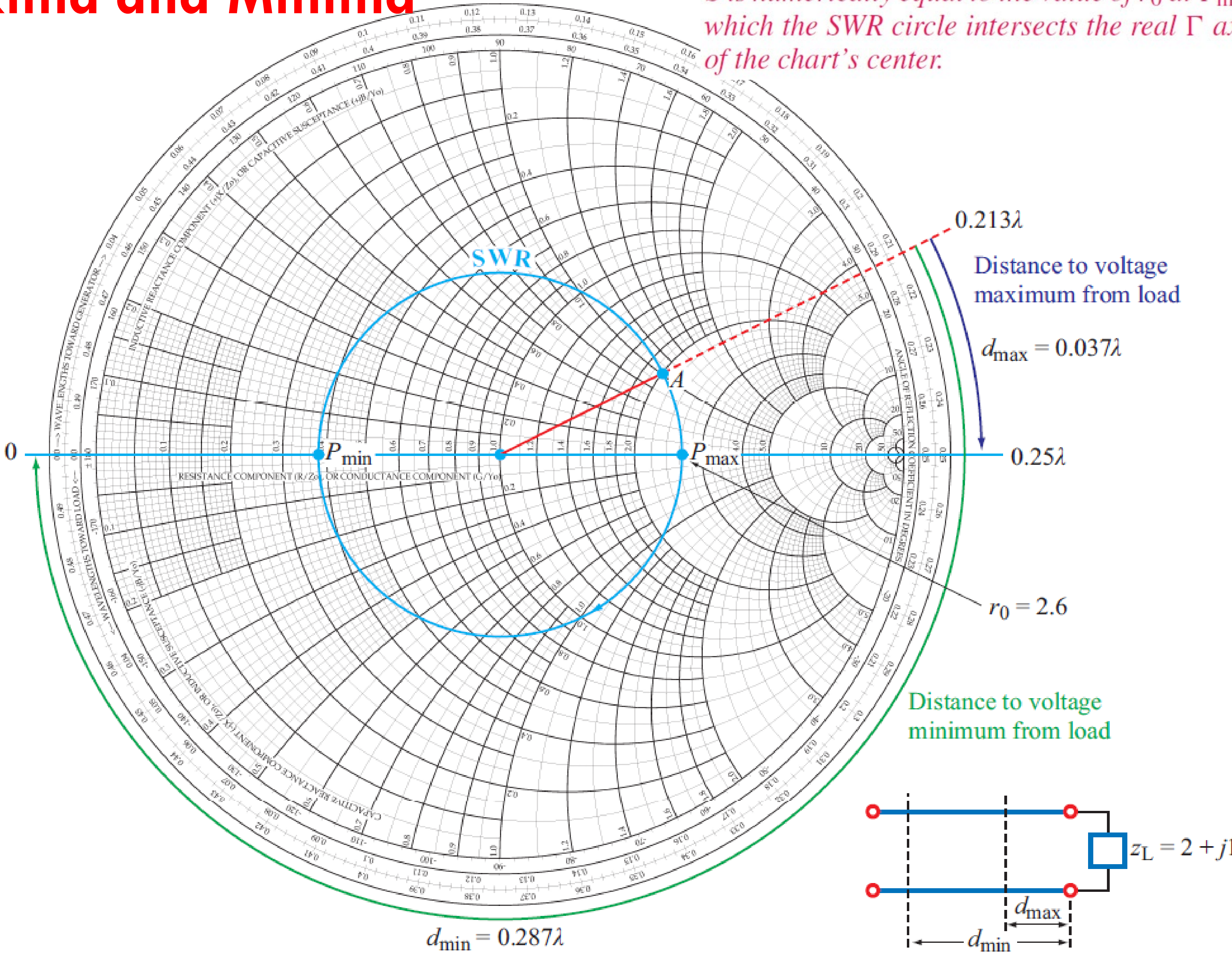
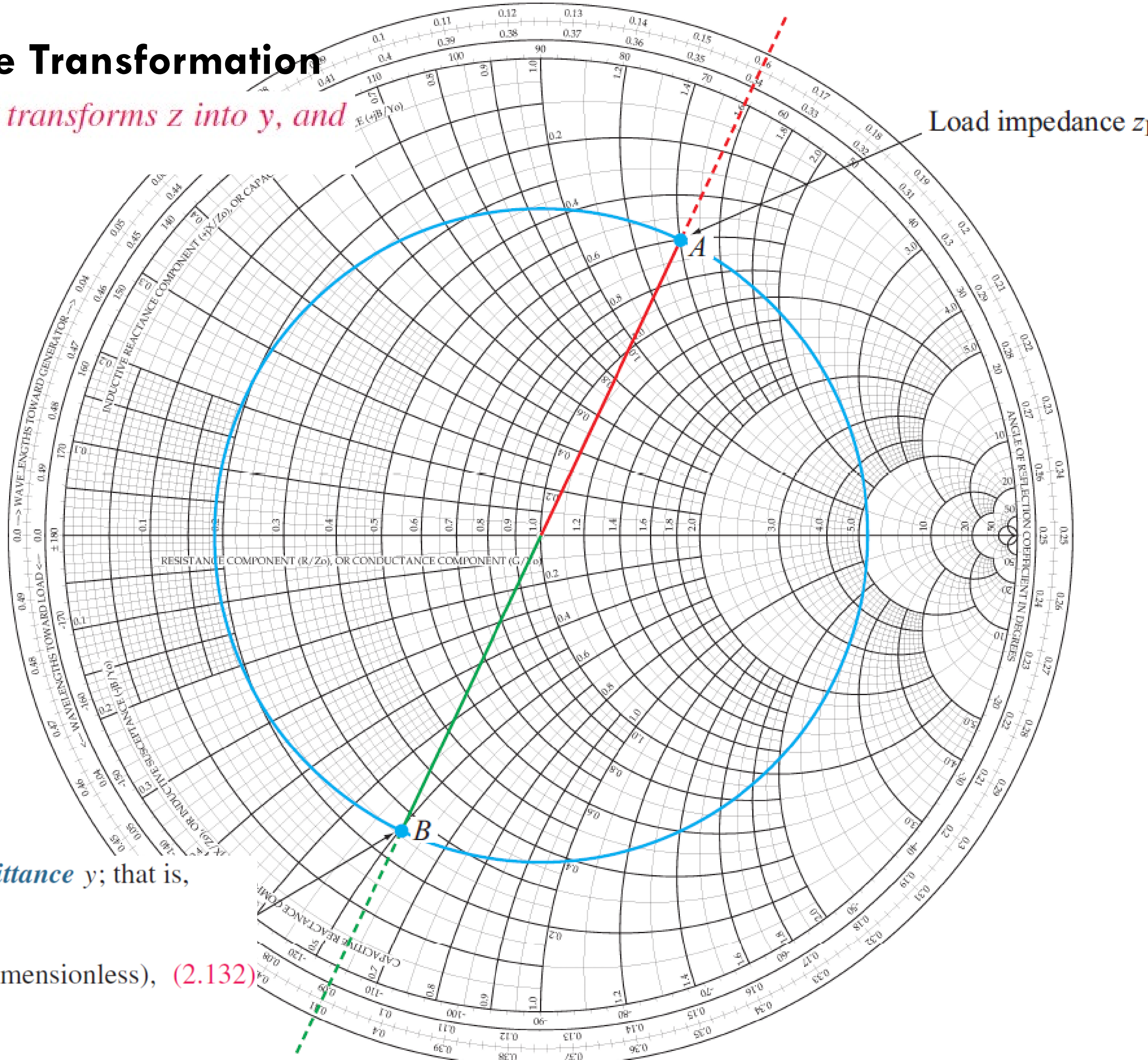


Figure 2-28: Point A represents a normalized load with $z_L = 2 + j1$. The standing wave ratio is $S = 2.6$ (at P_{max}), the distance between the load and the first voltage maximum is $d_{max} = (0.25 - 0.213)\lambda = 0.037\lambda$, and the distance between the load and the first voltage minimum is $d_{min} = (0.037 + 0.25)\lambda = 0.287\lambda$.

Impedance to Admittance Transformation

Rotation by λ/4 on the SWR circle transforms z into y, and vice versa.



to the definition of the ***normalized admittance*** y; that is,

$$y = \frac{Y}{Y_0} = \frac{G}{Y_0} + j\frac{B}{Y_0} = g + jb \qquad \text{(dimensionless)}, \quad (2.132)$$

where $Y_0 = 1/Z_0$ is the ***characteristic admittance of the line***

Example : Smith Chart Calculations

A 50-Ω lossless transmission line of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)\ \Omega$. Use the Smith

$$z_L = \frac{Z_L}{Z_0} = \frac{25 + j50}{50} = 0.5 + j1$$

(a)

$$d_{max} = (0.25 - 0.135)\lambda = 0.115\lambda$$

$$d_{min} = (0.5 - 0.135)\lambda = 0.365\lambda$$

(b)

$$z_{in} = 0.28 - j0.40$$

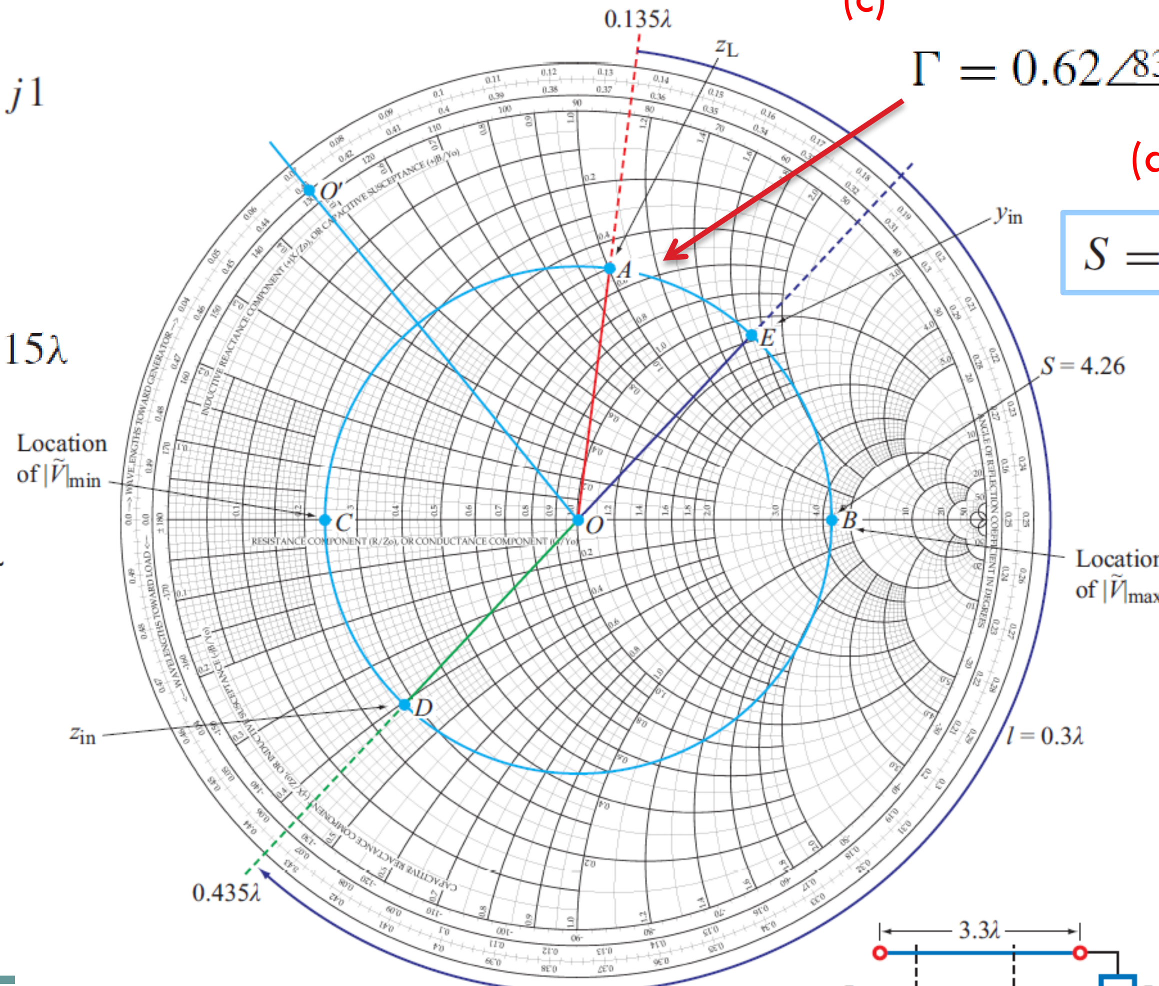
$$Z_{in} = z_{in} Z_0 = (0.28 - j0.40)50 = (14 - j20)\ \Omega$$

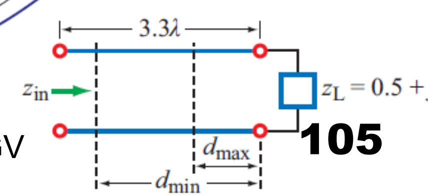
(c)

$$\Gamma = 0.62\angle 83^{\circ}$$

(d)

$$\boxed{S = 4.26}$$





Determining Z_L Using the Smith Chart

Given:

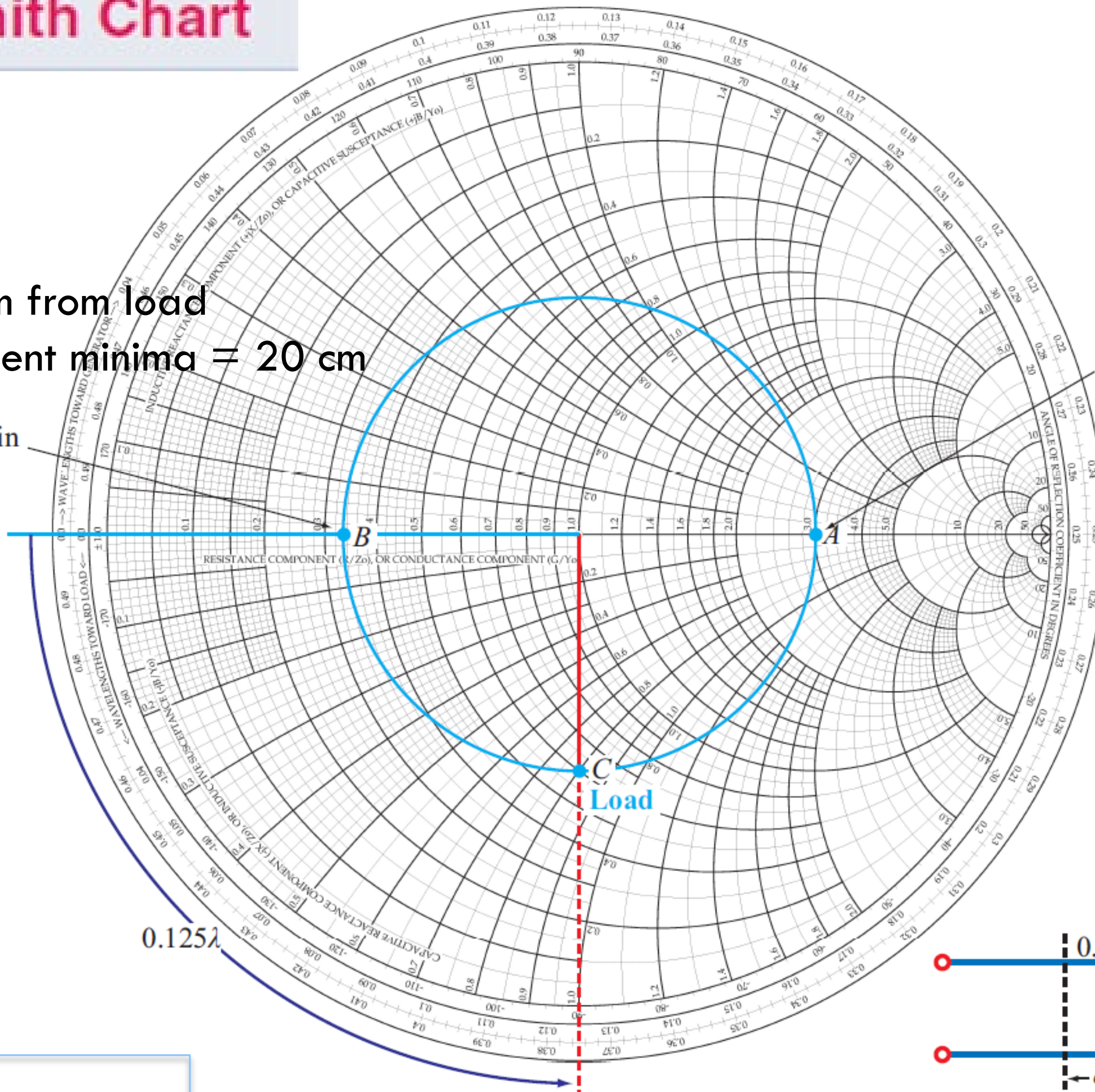
$S = 3$

$Z_0 = 50\ \Omega$

first voltage min @ 5 cm from load

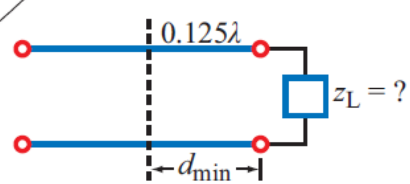
Distance between adjacent minima = 20 cm

Determine: Z_L



$$d_{\min} = \frac{5}{40} = 0.125\lambda$$

$$z_L = 0.6 - j0.8$$



$$Z_L = 50(0.6 - j0.8) = \boxed{(30 - j40)\ \Omega}$$

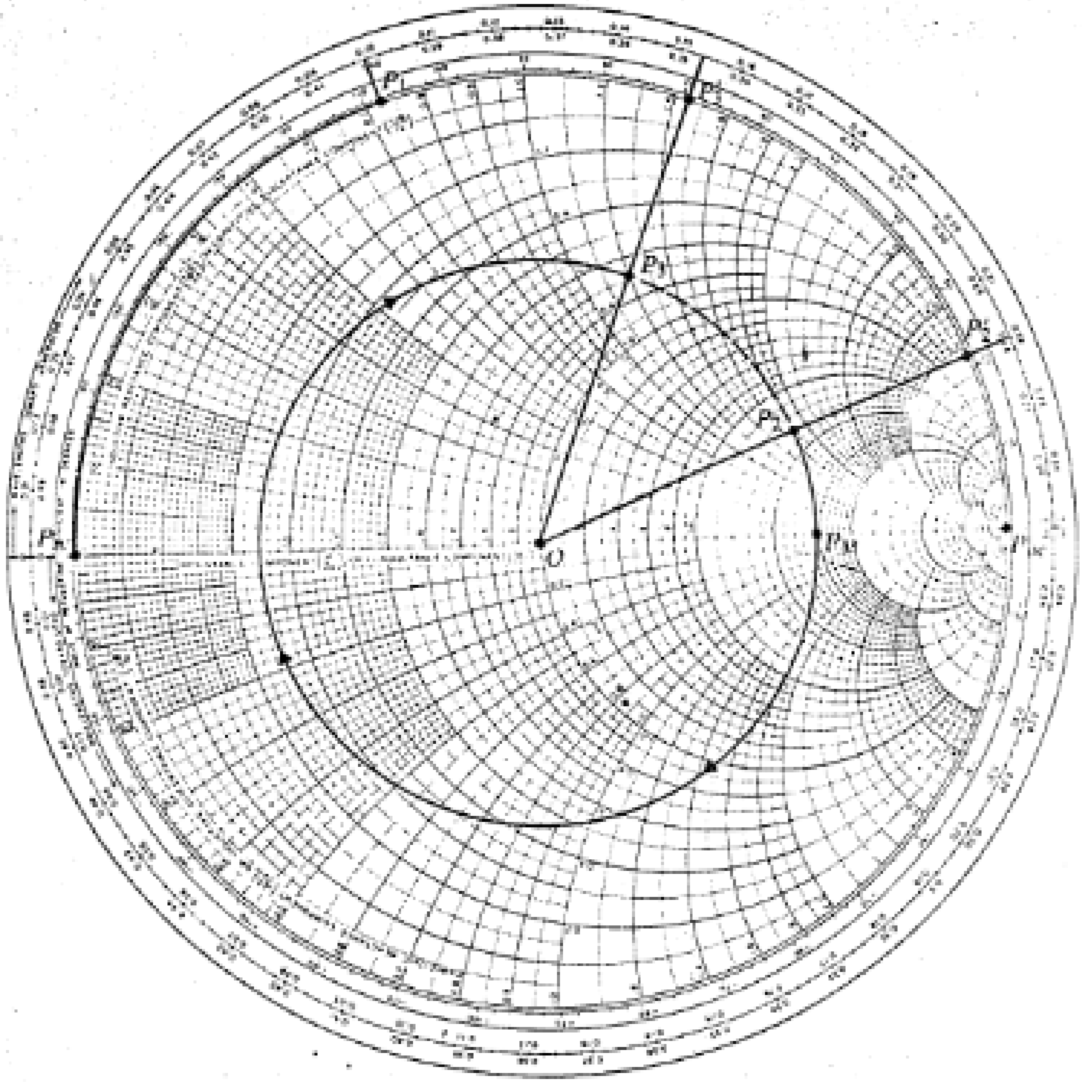
SMITH CHART

Example 9–9 Use the Smith chart to find the input impedance of a section of a 50-(Ω) lossless transmission line which is 0.1 wavelength long and is terminated in a short-circuit.

Solution: Given: $z_L = 0 \quad R_0 = 50\ (\Omega) \quad d = 0.1\lambda.$

1. Enter the Smith chart at the intersection of $r = 0$ and $x = 0$ (Point P_{sc} on the extreme left of chart. See Fig. 9–16.)
2. Move along the perimeter of the chart ($|\Gamma| = 1$) by 0.1 "wavelengths toward generator" in a clockwise direction to P_1.
3. At P_1 read $r = 0$ and $x \cong 0.725$, or $z_i = j0.725$. Thus, $Z_i = R_0 z_i = 50(j0.725) = j36.3\ (\Omega)$. (The input impedance is purely inductive.)

SMITH CHART



SMITH CHART

Example 9-1[illegible] A lossless transmission line of length 0.434λ and characteristic impedance 100 (Ω) is terminated in an impedance 260 + j180 (Ω). Find (a) the voltage reflection coefficient, (b) the standing-wave ratio, (c) the input impedance, and (d) the location of a voltage maximum on the line.

Solution: Given that d=0.434λ, Z_0=100 Ω, Z_L=260+j180 Ω.
Therefore

$$\bar{z}_l = \frac{Z_l}{Z_0} = 2.6 + j1.8$$ (P_2 on the chart)

$OP_2 = |\Gamma_l| = 0.6$

Extend OP_2 to P_2' and 0.220 on the wavelength scale.

Therefore $\theta_l = (0.25-0.22)4\pi = 21^0$. Then, $\Gamma_l = |\Gamma_l| \angle \theta_l = 0.6 \angle 21^0$. The $|\Gamma_l|$ circle intersects the real axis at r=4, thus ρ=4.

SMITH CHART

Move to P3 to find the input impedance which is 0.434λ far from load position towards the generator. The input impedance is 100(0.69+j1.2)=69+j120Ω.

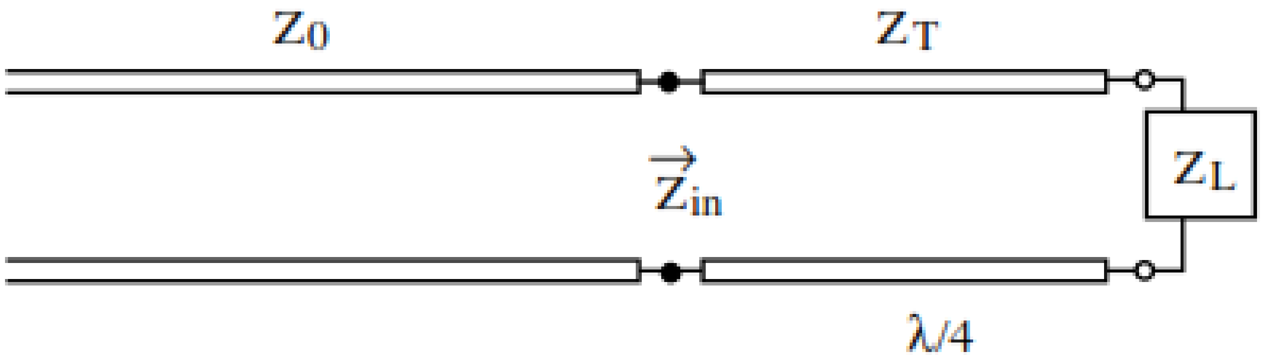
In moving from P2 to P3, the $|\Gamma_l|$ circle intersects the real axis at P_M where the voltage is maximum. Thus the voltage maximum appears at (0.25-0.22)λ =0.03 λ length away from the load towards the generator.

Impedance Matching

IMPEDANCE MATCHING

1) Quarter Wavelength Transformer

For Real Load



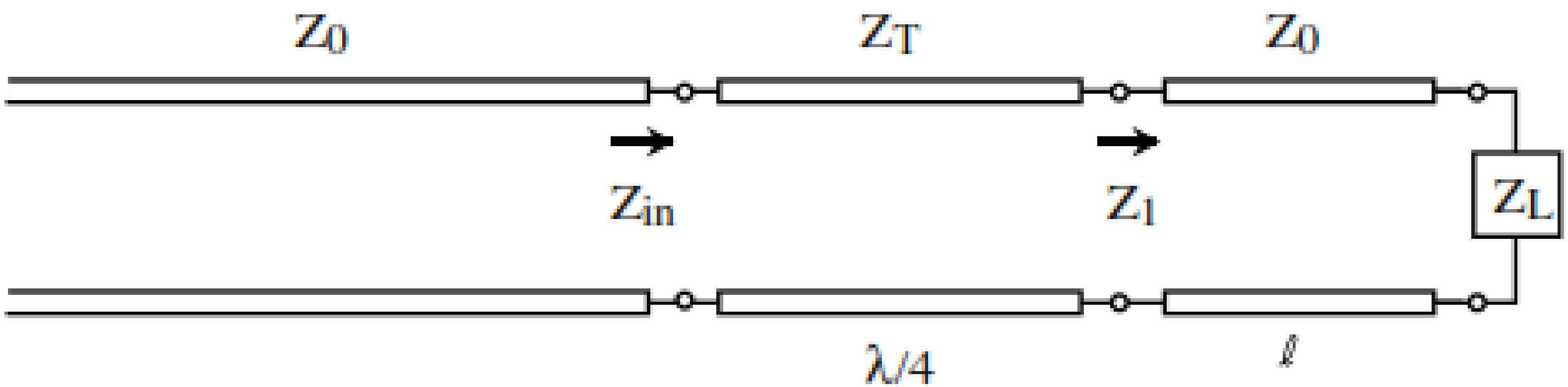
$$Z_T = \sqrt{Z_0 Z_l}\,.$$

IMPEDANCE MATCHING

1) Quarter Wavelength Transformer

For Complex Load

We want Z_1 to be real and $Z_{in}=Z_0$ for Z_T to be real and matching condition is satisfied.



IMPEDANCE MATCHING

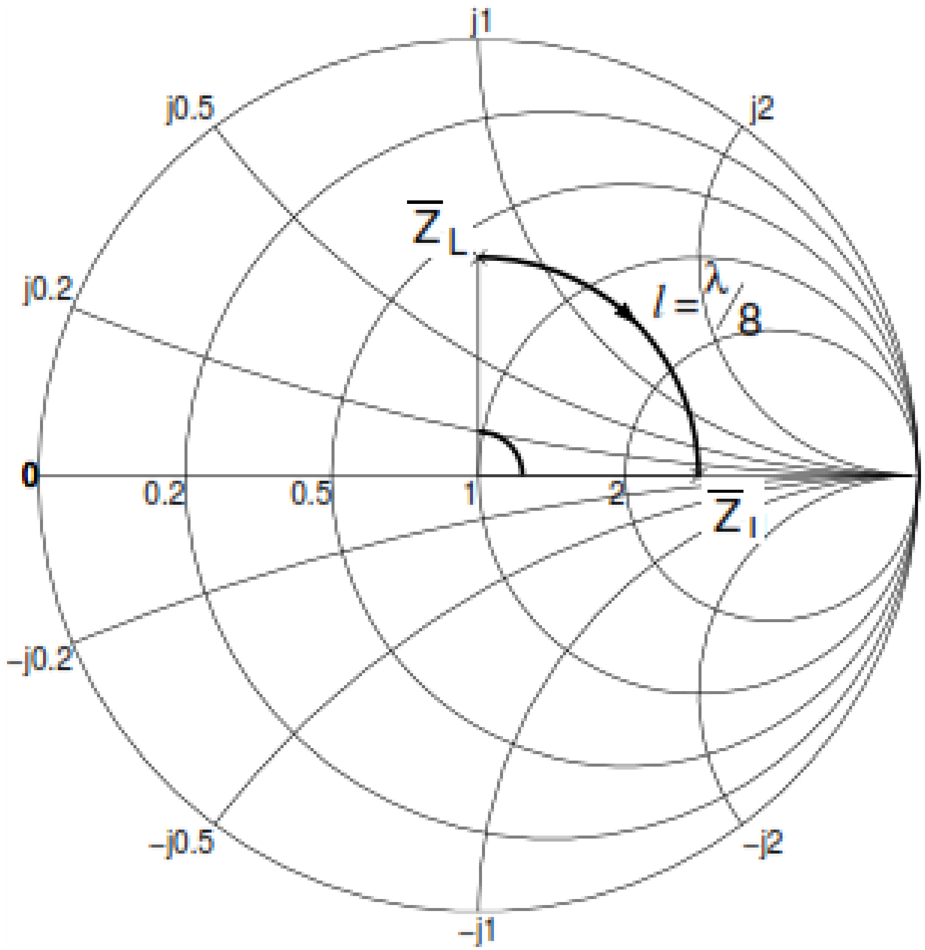
Given that $Z_L = (30 + j40)\Omega$, $Z_0 = 50\Omega$, find the shortest l and Z_T so that the above circuit is matched. Assume that Z_T is real and lossless.

We find that $Z_{nL} = 0.6 + j0.8$. In order to make Z_{n1} real, the shortest l from the Smith Chart is $\frac{\lambda}{8}$. Then $Z_{n1} = 3.0$, and $Z_1 = 150\Omega$. Since $Z_{in} = 50\Omega$, we need

$$Z_T = \sqrt{Z_{in} Z_1} = \sqrt{50 \times 150} = 86.6\Omega$$

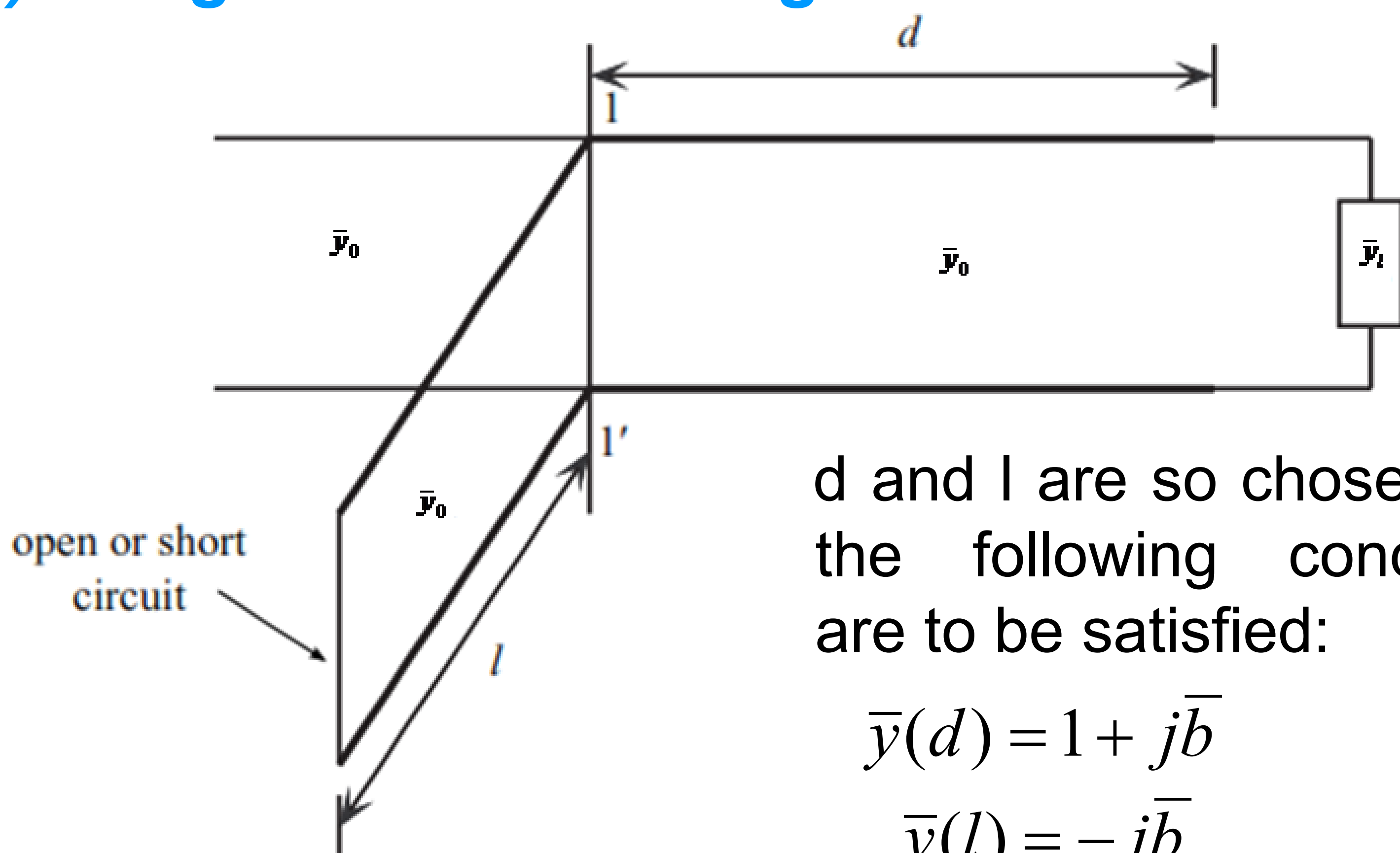
in order for matching condition to be satisfied.

IMPEDANCE MATCHING



IMPEDANCE MATCHING

1) Single Stub Matching



d and l are so chosen that the following conditions are to be satisfied:

$$\bar{y}(d) = 1 + j\bar{b}$$

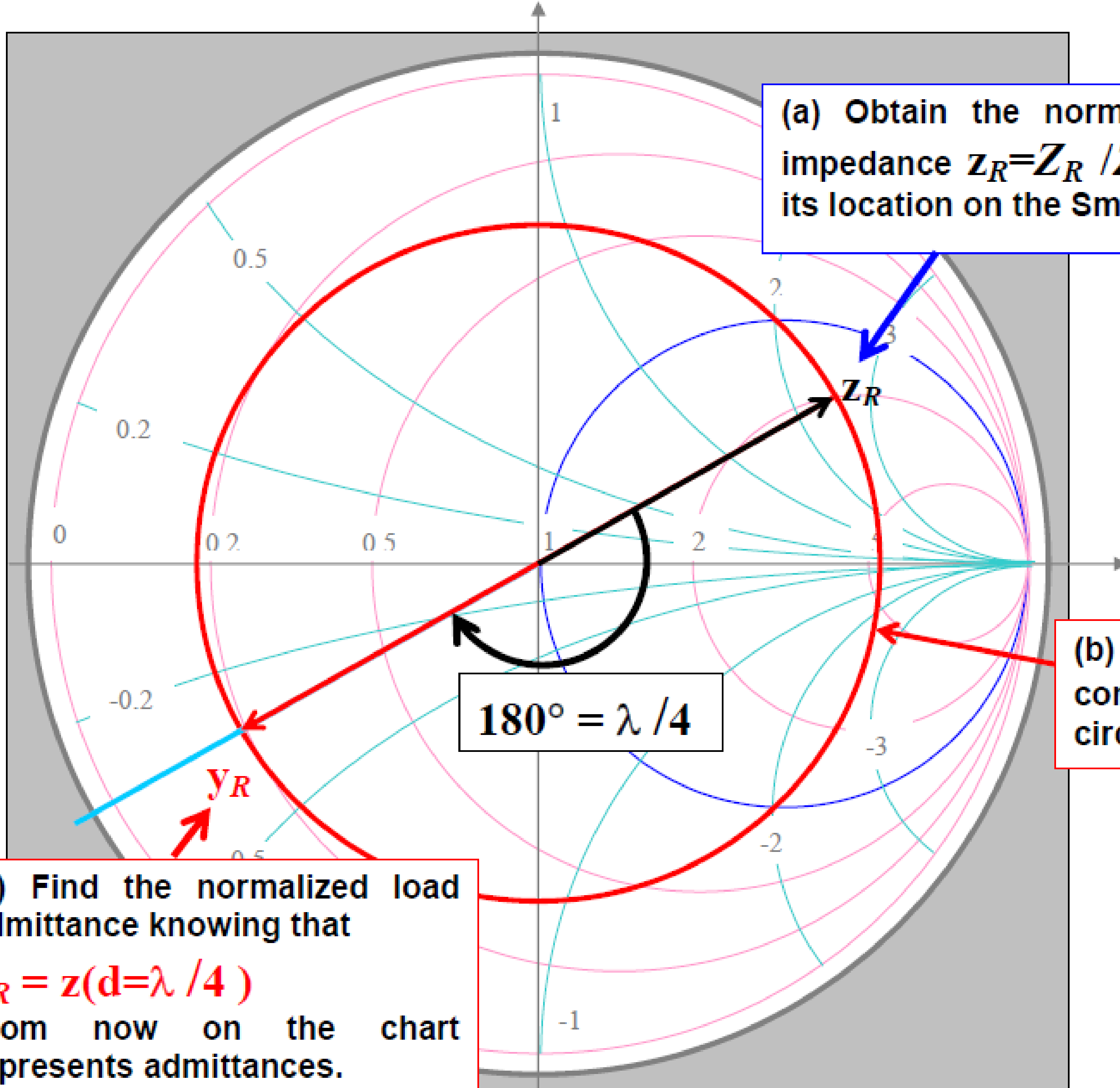
$$\bar{y}(l) = -j\bar{b}$$

Single stub matching problems can be solved on the Smith chart graphically, using a compass and a ruler. This is a step-by-step summary of the procedure:

(a) Find the normalized load impedance and determine the corresponding location on the chart.

(b) Draw the circle of constant magnitude of the reflection coefficient $|\Gamma|$ for the given load.

(c) Determine the normalized load admittance on the chart. This is obtained by rotating 180° on the constant $|\Gamma|$ circle, from the load impedance point. From now on, all values read on the chart are normalized admittances.



(a) Obtain the normalized load impedance $z_R = Z_R / Z_0$ and find its location on the Smith chart

(b) Draw the constant $|\Gamma(d)|$ circle

(c) Find the normalized load admittance knowing that

$y_R = z(d = \lambda/4)$

From now on the chart represents admittances.

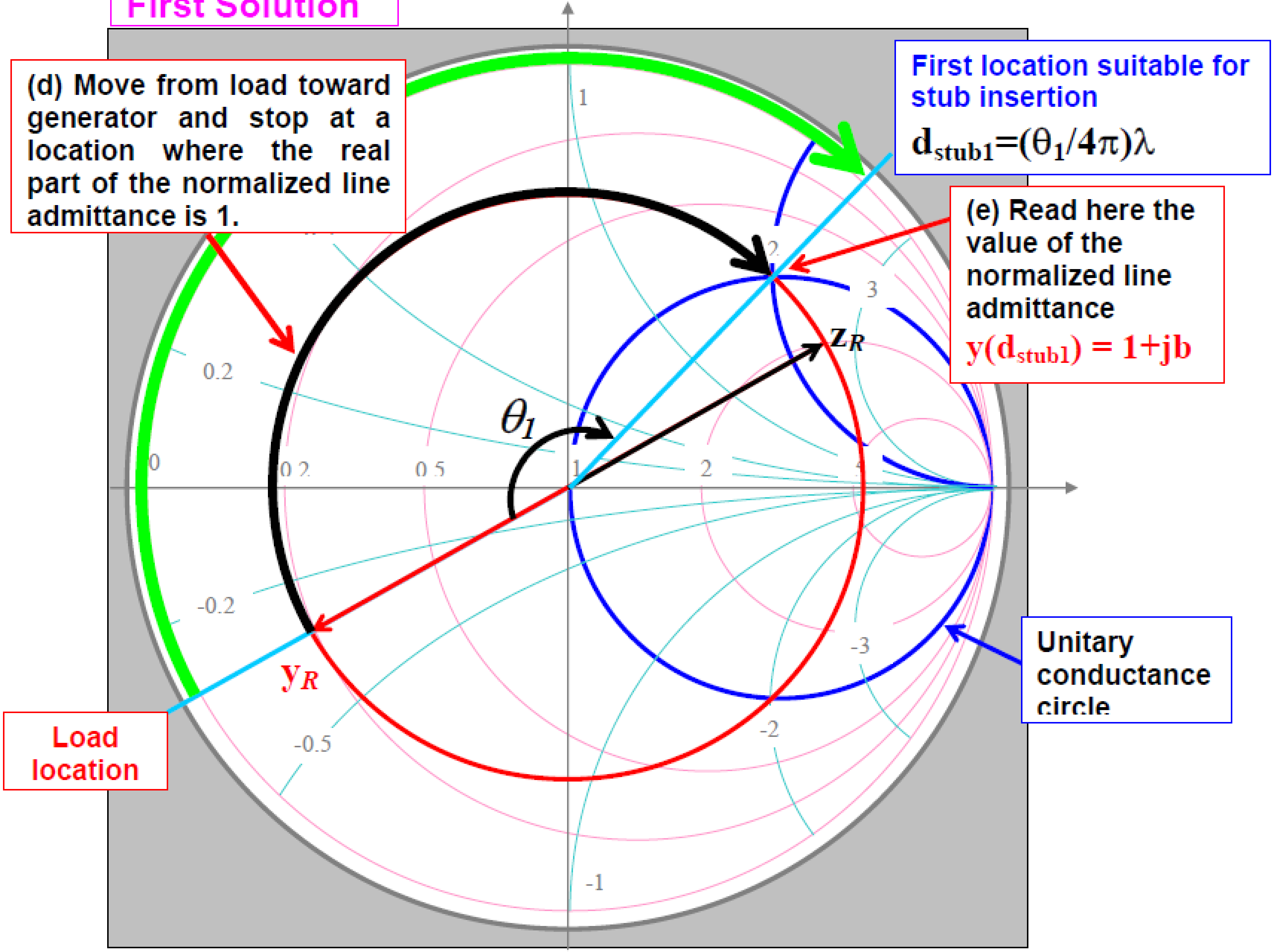
(d) Move from load admittance toward generator by riding on the constant $|\Gamma|$ circle, until the intersections with the unitary normalized conductance circle are found. These intersections correspond to possible locations for stub insertion. Commercial Smith charts provide graduations to determine the angles of rotation as well as the distances from the load in units of wavelength.

(e) Read the line normalized admittance in correspondence of the stub insertion locations determined in (d). These values will always be of the form

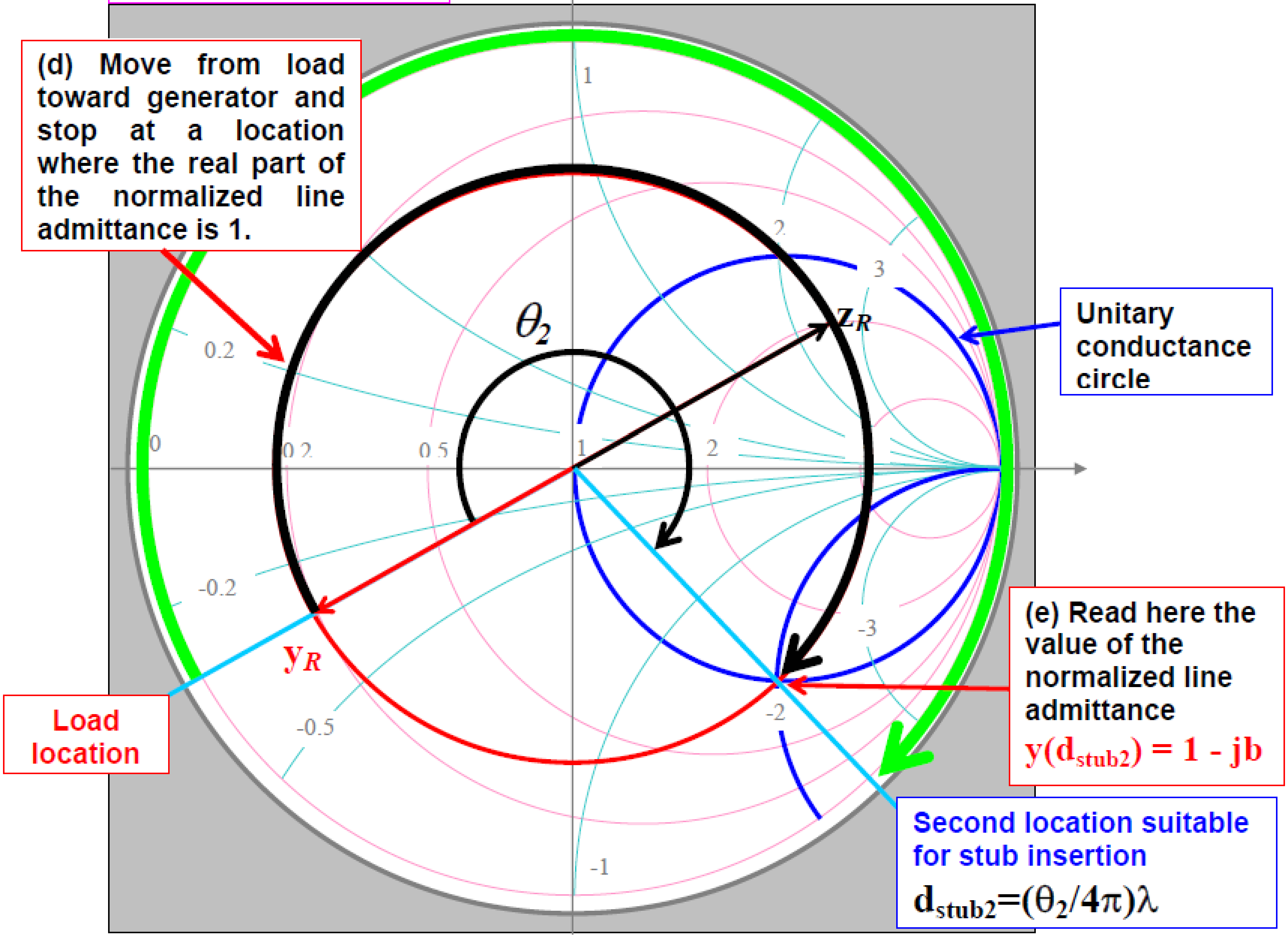
$$y(d_{stub}) = 1 + jb \qquad \text{top half of chart}$$

$$y(d_{stub}) = 1 - jb \qquad \text{bottom half of chart}$$

First Solution



Second Solution



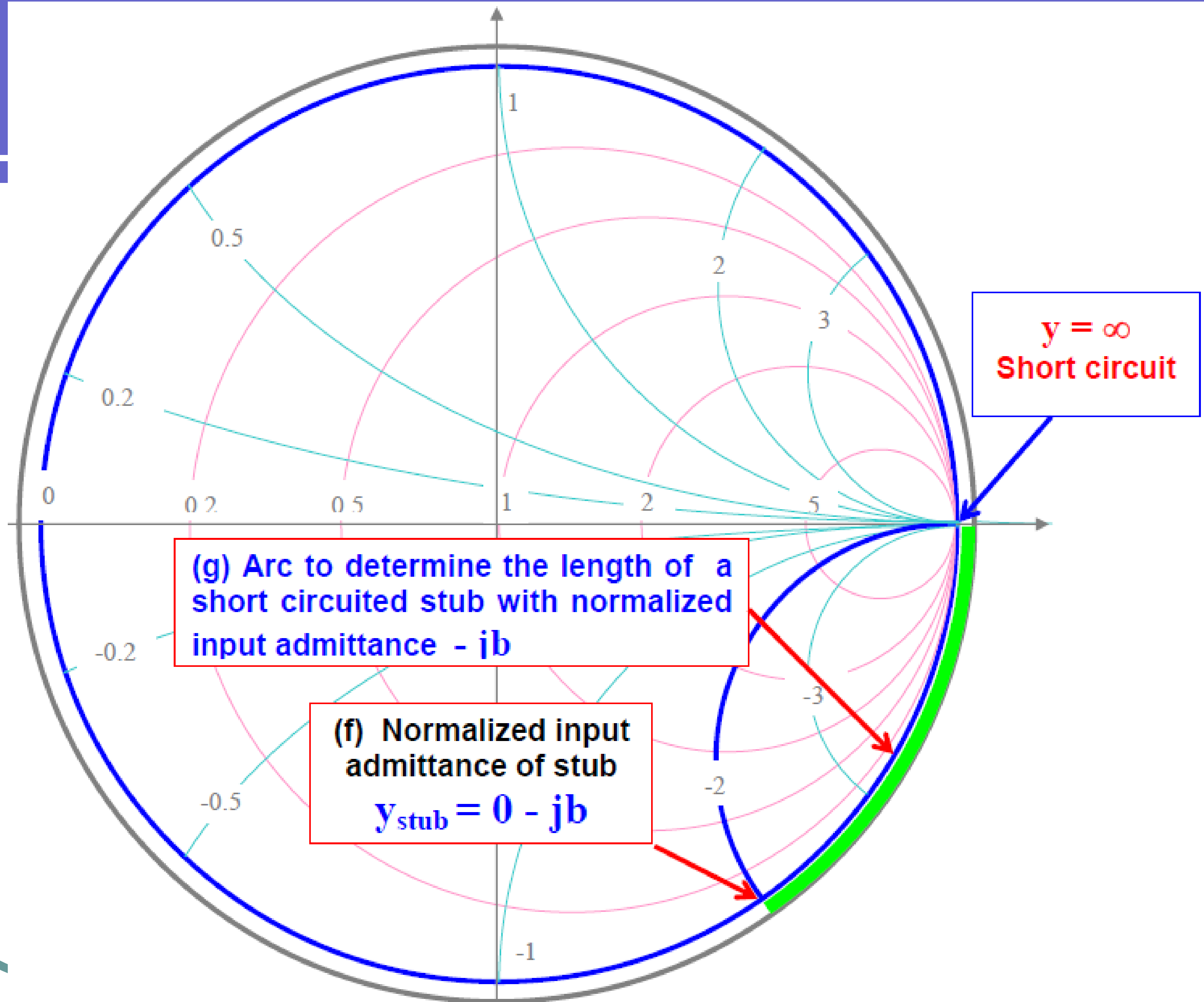
(f) Select the input normalized admittance of the stubs, by taking the opposite of the corresponding imaginary part of the line admittance

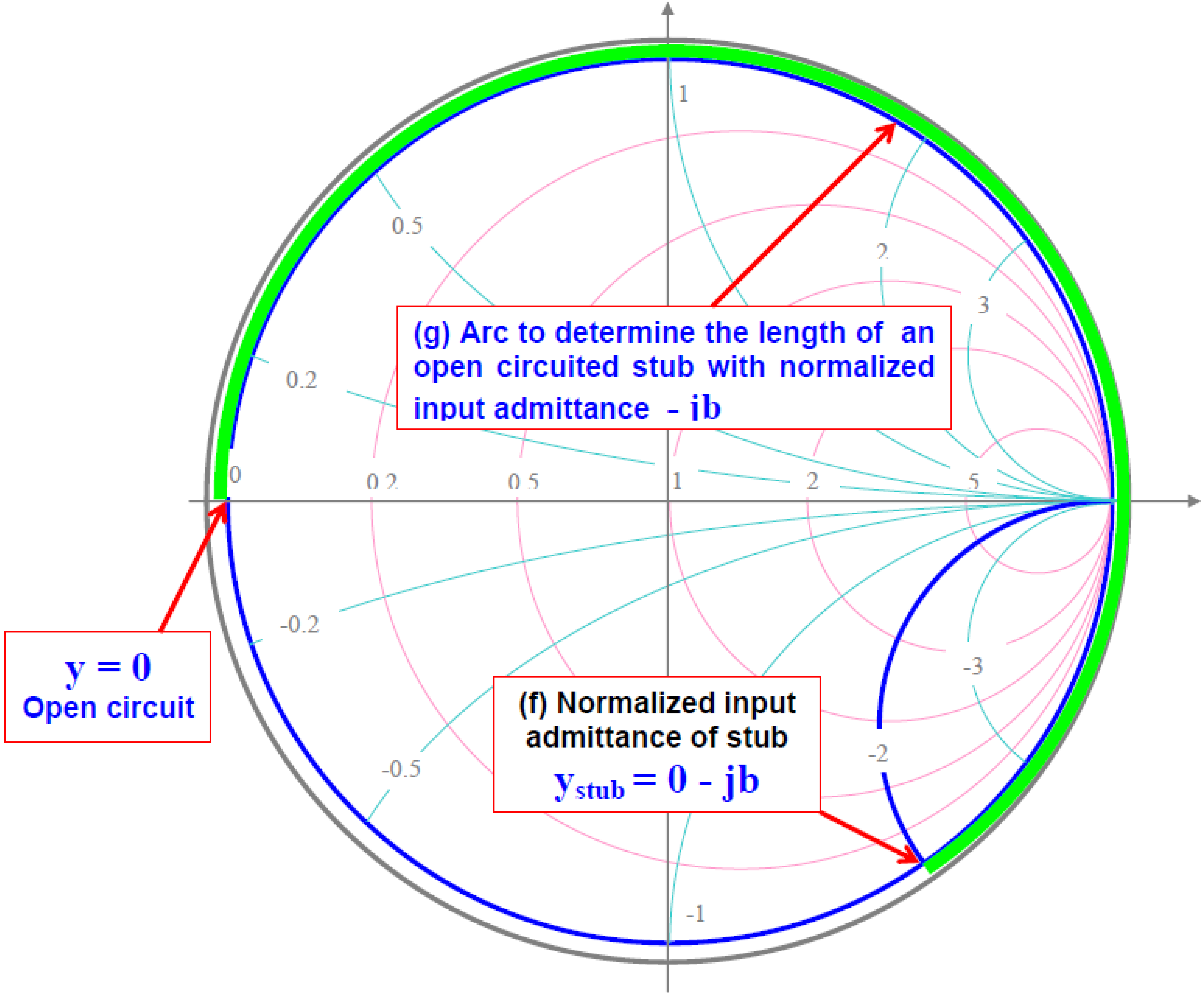
$$\text{line: } y(\mathrm{d}_{\mathrm{stub}}) = 1 + jb \qquad \rightarrow \qquad \text{stub: } y_{\mathrm{stub}} = -jb$$

$$\text{line: } y(\mathrm{d}_{\mathrm{stub}}) = 1 - jb \qquad \rightarrow \qquad \text{stub: } y_{\mathrm{stub}} = +jb$$

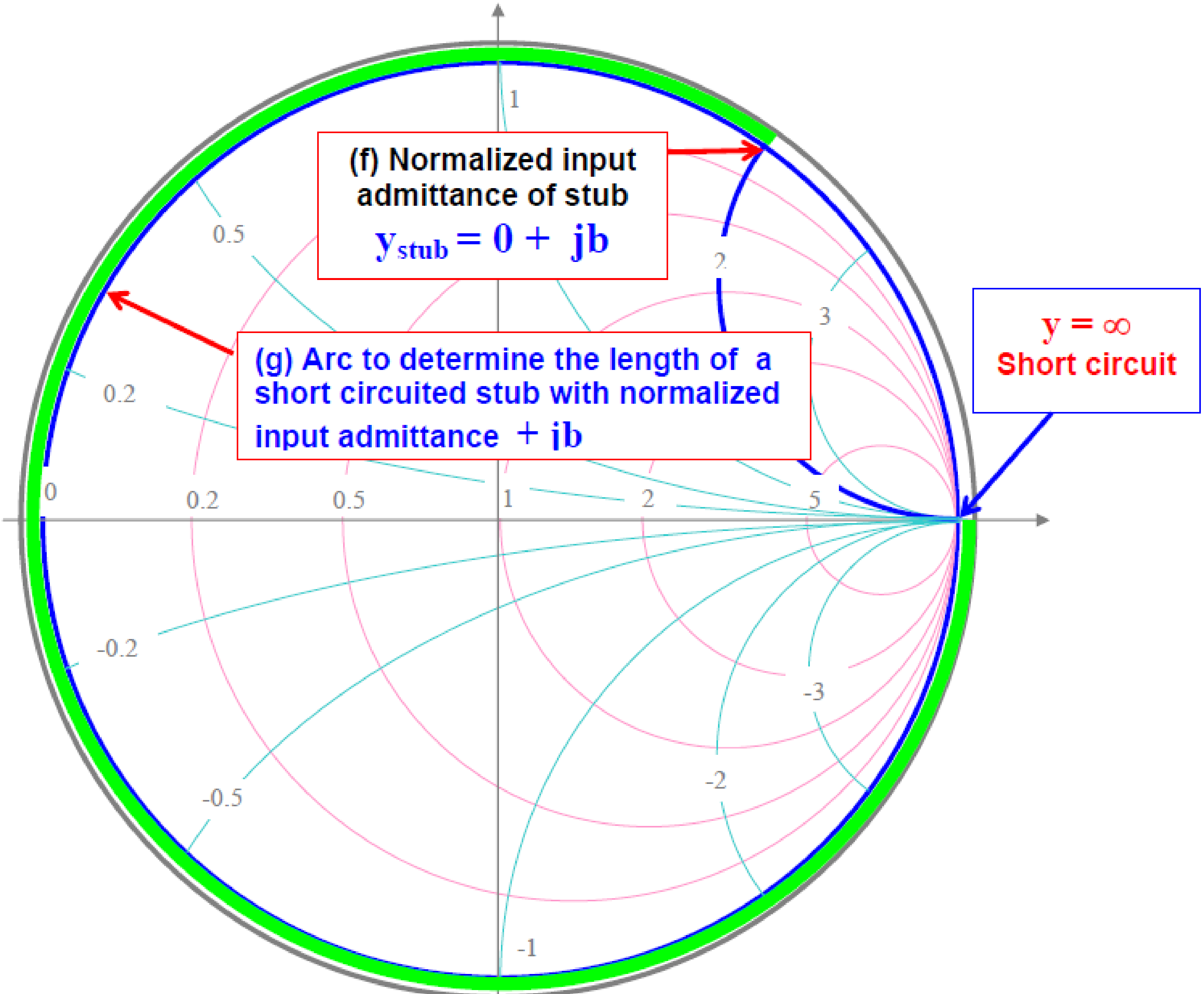
(g) Use the chart to determine the length of the stub. The imaginary normalized admittance values are found on the circle of zero conductance on the chart. On a commercial Smith chart one can use a printed scale to read the stub length in terms of wavelength. We assume here that the stub line has characteristic impedance Z_0 as the main line. If the stub has characteristic impedance $Z_{0S} \neq Z_0$ the values on the Smith chart must be renormalized as

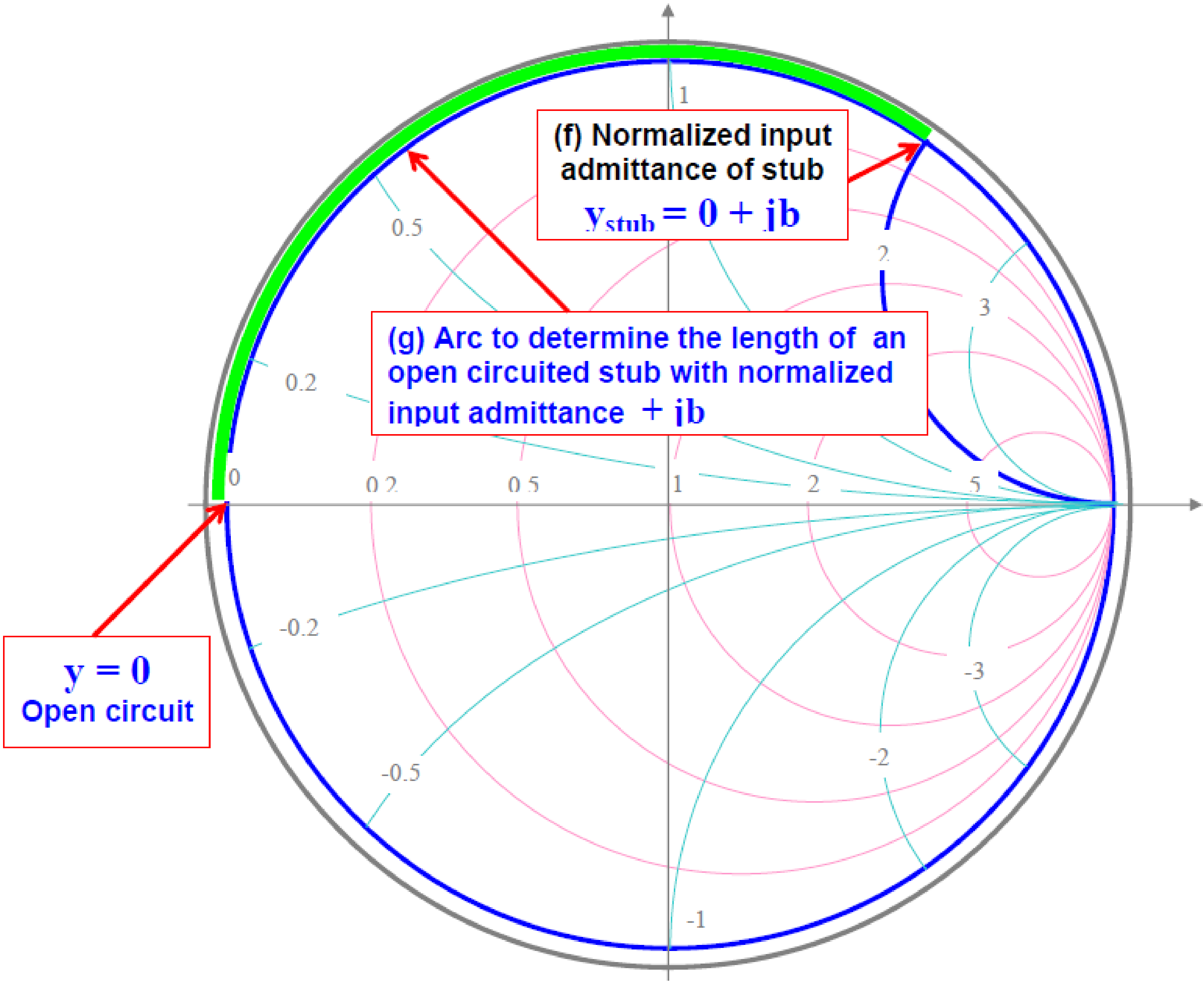
$$\pm jb' = \pm jb \frac{Y_0}{Y_{0s}} = \pm jb \frac{Z_{0s}}{Z_0}$$


1
0.5
2
3
0.2
0
0.2
0.5
1
2
5
y = ∞
Short circuit
(g) Arc to determine the length of a short circuited stub with normalized input admittance - jb
-0.2
-3
(f) Normalized input admittance of stub
$y_{stub} = 0 - jb$
-0.5
-2
-1



1
0.5
2
3
0.2
(g) Arc to determine the length of an open circuited stub with normalized input admittance - jb
0
0.2
0.5
1
2
5
-0.2
y = 0
Open circuit
-3
-0.5
(f) Normalized input admittance of stub
$y_{stub} = 0 - jb$
-2
-1


(f) Normalized input admittance of stub
$y_{stub} = 0 + jb$
(g) Arc to determine the length of a short circuited stub with normalized input admittance $+ jb$
$y = \infty$
Short circuit
1
0.5
0.2
0
0.2
0.5
1
2
5
2
3
-0.2
-0.5
-1
-2
-3



(f) Normalized input admittance of stub
$y_{stub} = 0 + jb$
(g) Arc to determine the length of an open circuited stub with normalized input admittance $+ jb$
$y = 0$
Open circuit
1
0.5
0.2
2
3
0
0.2
0.5
1
2
5
-0.2
-0.5
-3
-2
-1

Week 7

Slide 128-147

SINGLE STUB MATCHING-EXAMPLE

Example 9–16 A 50-(Ω) transmission line is connected to a load impedance $Z_L = 35 - j47.5$ (Ω). Find the position and length of a short-circuited stub required to match the line.

Given

$$R_0 = 50\ (\Omega)$$
$$Z_L = 35 - j47.5\ (\Omega)$$
$$z_L = Z_L/R_0 = 0.70 - j0.95. \qquad P_1$$

SINGLE STUB MATCHING- EXAMPLE

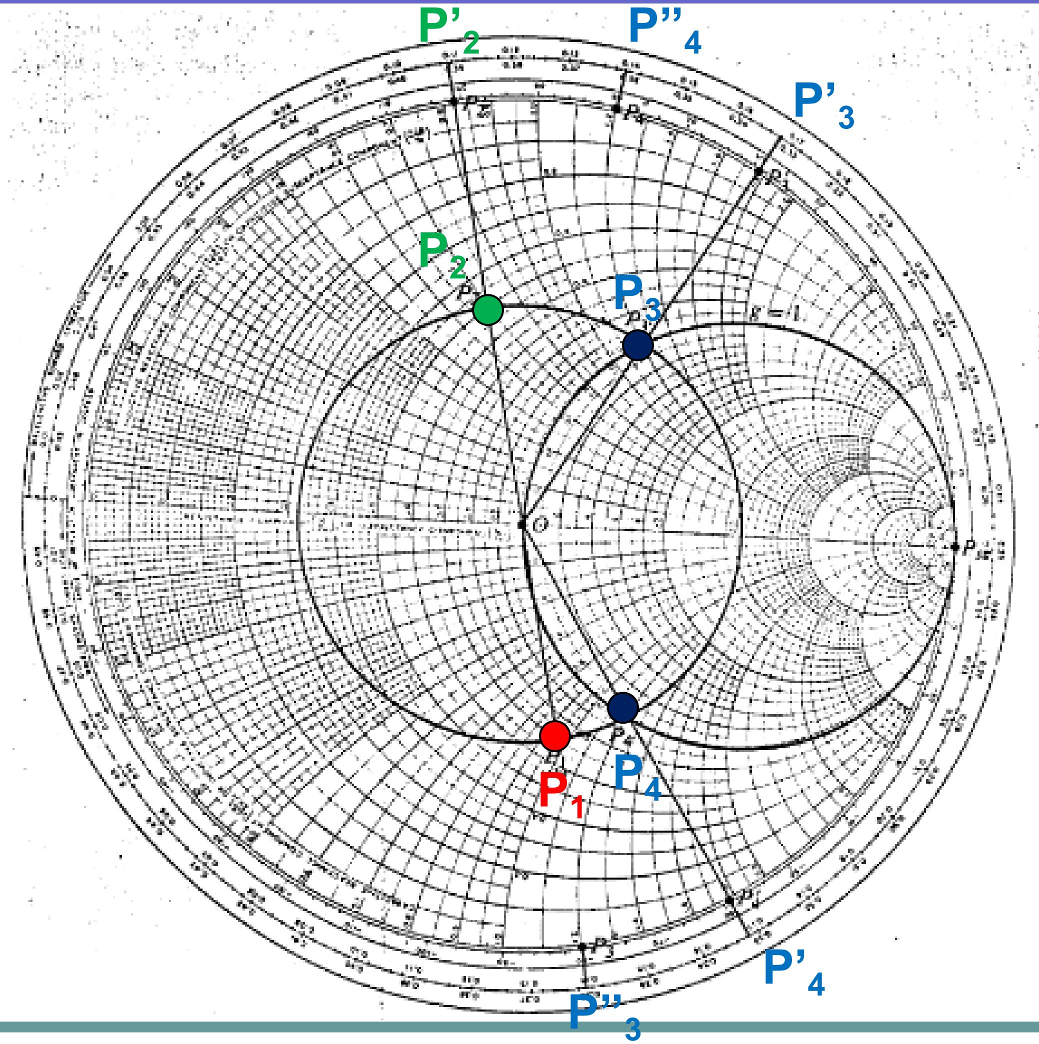
1. Enter z_L on the Smith chart as P_1 (Fig. 9-23).
2. Draw a $|\Gamma|$-circle centered at O with radius $\overline{OP_1}$.
3. Draw a straight line from P_1 through O to point P_2' on the perimeter, intersecting the $|\Gamma|$-circle at P_2, which represents y_L. Note 0.109 at P_2' on the "wavelengths toward generator" scale.
4. Note the two points of intersection of the $|\Gamma|$-circle with the $g = 1$ circle.

$$\text{At } P_3\text{:} \quad y_{B1} = 1 + j1.2 = 1 + jb_{B1};$$
$$\text{At } P_4\text{:} \quad y_{B2} = 1 - j1.2 = 1 + jb_{B2}.$$

5. Solutions for the position of the stub:

$$\text{For } P_3 \text{ (from } P_2' \text{ to } P_3'\text{):} \quad d_1 = (0.168 - 0.109)\lambda = 0.059\lambda;$$
$$\text{For } P_4 \text{ (from } P_2' \text{ to } P_4'\text{):} \quad d_2 = (0.332 - 0.109)\lambda = 0.223\lambda.$$

SINGLE STUB MATCHING- EXAMPLE



SINGLE STUB MATCHING- EXAMPLE

6. Solutions for the length of short-circuited stub to provide $y_s = -jb_B$:

For P_3 (from P_{sc} on the extreme right of chart to P_3'', which represents $-jb_{B1} = -j1.2$):

$$\ell_{B1} = (0.361 - 0.250)\lambda = 0.111\lambda;$$

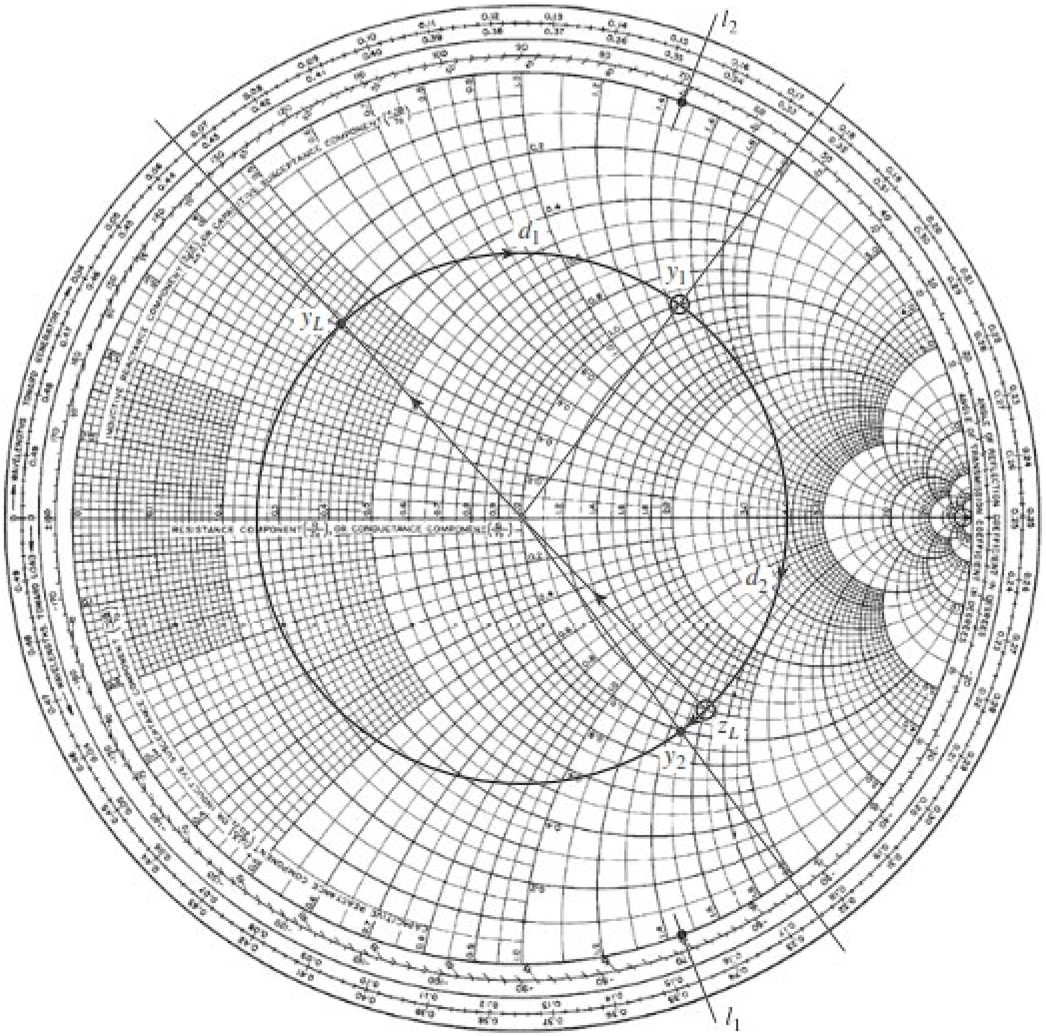
For P_4 (from P_{sc} to P_4'', which represents $-jb_{B2} = j1.2$):

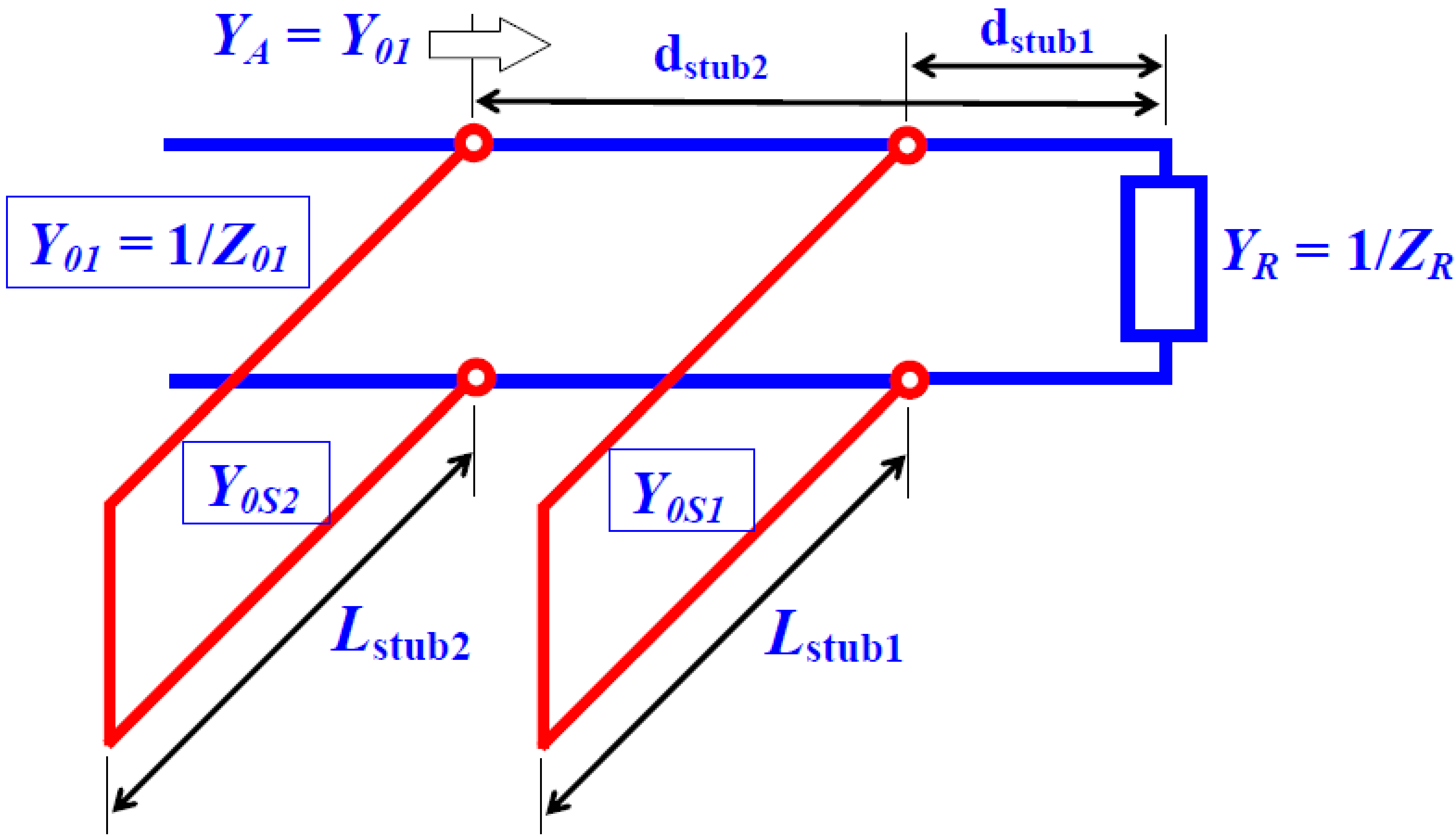
$$\ell_{B2} = (0.139 + 0.250)\lambda = 0.389\lambda.$$

SINGLE STUB MATCHING

For a load impedance $Z_L = 60 - j80\ \Omega$, design two single-stub (short circuit) shunt tuning networks to match this load to a 50 Ω line. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and capacitor in series, plot the reflection coefficient magnitude from 1 to 3 GHz for each solution.

SINGLE STUB MATCHING





$Y_A = Y_{01}$
d_{stub2}
d_{stub1}
$Y_{01} = 1/Z_{01}$
$Y_R = 1/Z_R$
Y_{0S2}
Y_{0S1}
L_{stub2}
L_{stub1}

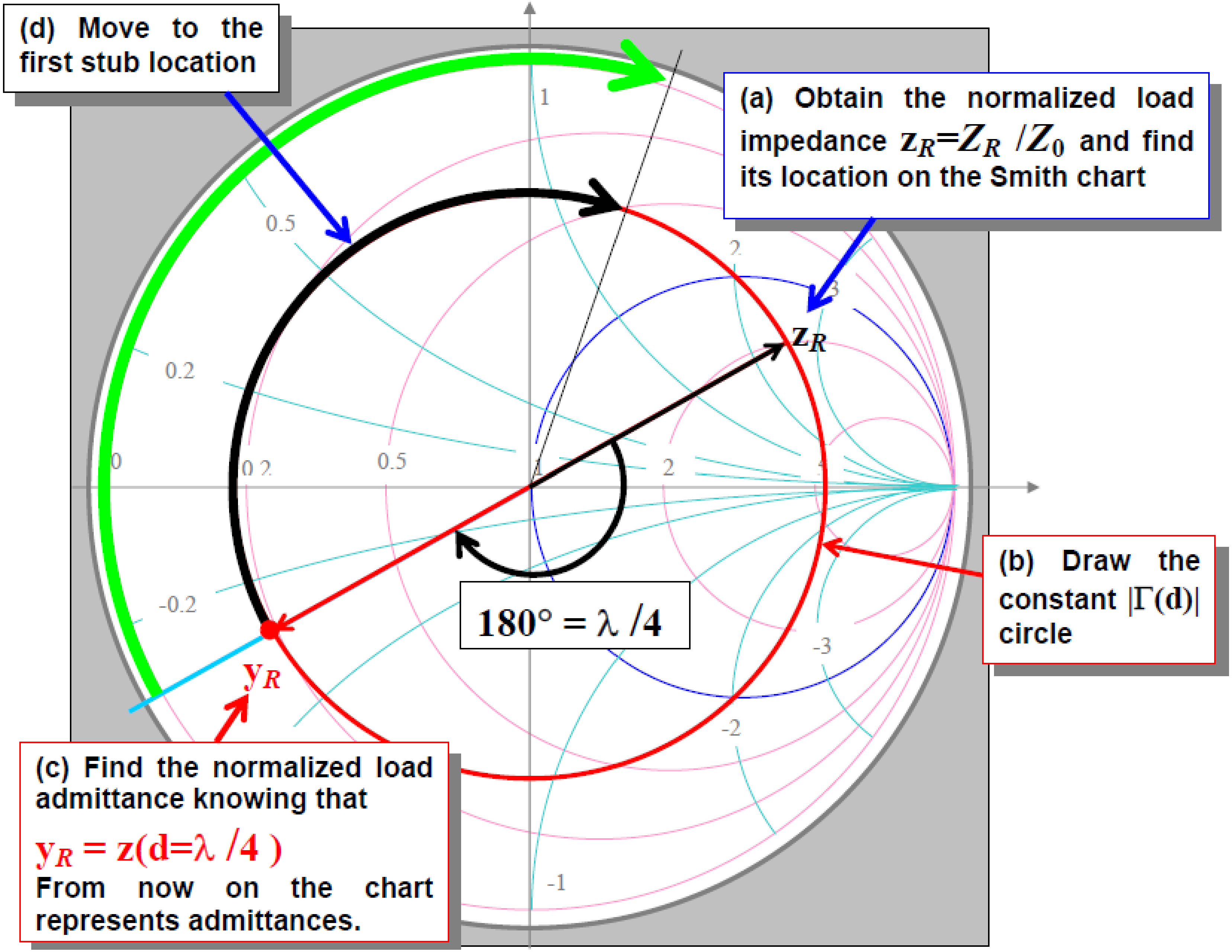
Given the load impedance, we need to follow these steps to complete the double stub design:

(a) Find the normalized load impedance and determine the corresponding location on the chart.

(b) Draw the circle of constant magnitude of the reflection coefficient $|\Gamma|$ for the given load.

(c) Determine the normalized load admittance on the chart. This is obtained by rotating -180° on the constant $|\Gamma|$ circle, from the load impedance point. From now on, all values read on the chart are normalized admittances.

(d) Find the normalized admittance at location d_{stub1} by moving clockwise on the constant $|\Gamma|$ circle.

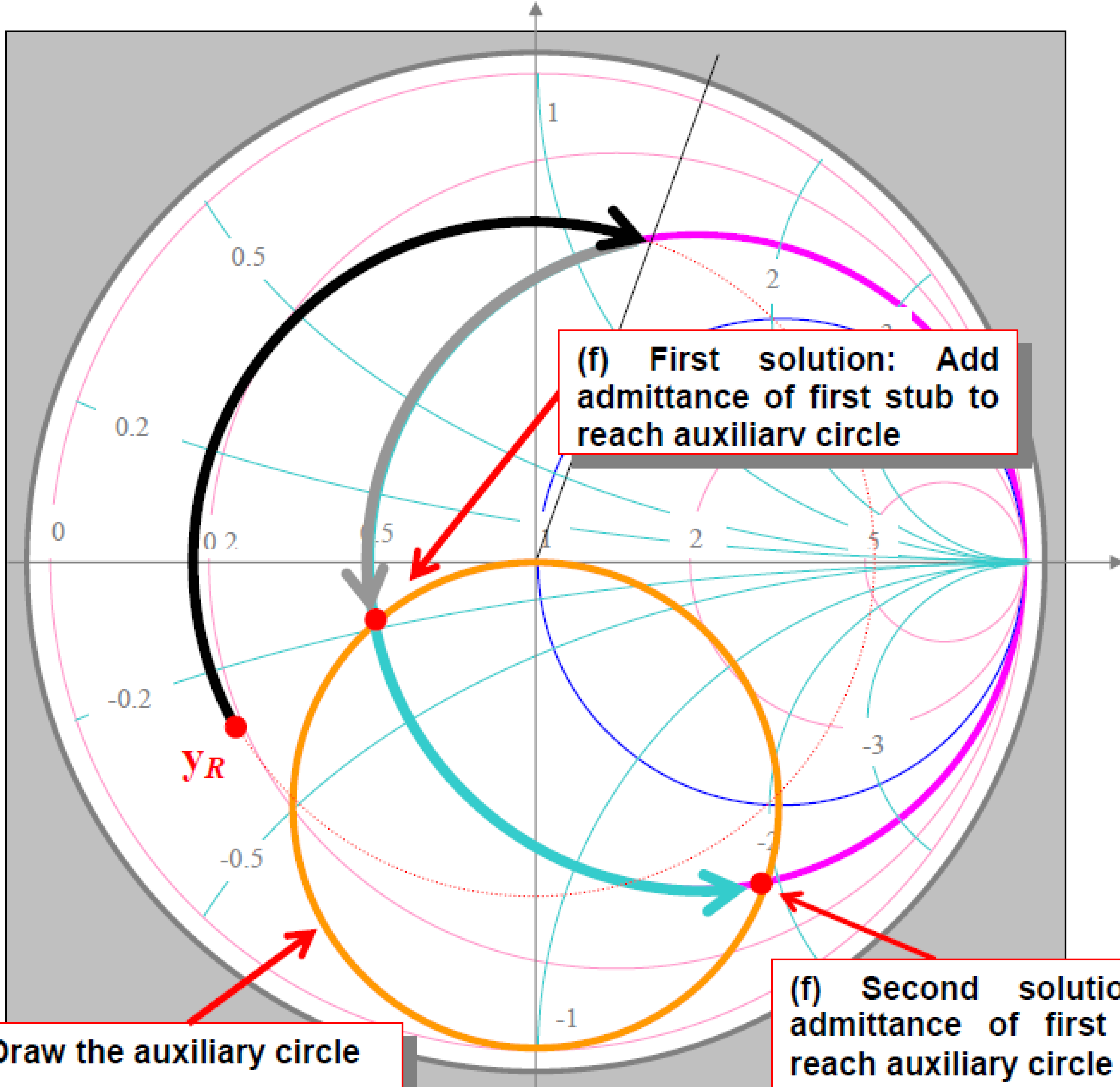

(d) Move to the first stub location
(a) Obtain the normalized load impedance $z_R = Z_R / Z_0$ and find its location on the Smith chart
z_R
(b) Draw the constant $|\Gamma(d)|$ circle
$180° = \lambda / 4$
y_R
(c) Find the normalized load admittance knowing that
$y_R = z(d = \lambda / 4)$
From now on the chart represents admittances.
0.5
0.2
0
0.2
0.5
1
2
-0.2
-3
-2
-1
1
2
3
4

(e) Draw the auxiliary circle

(f) Add the first stub admittance so that the normalized admittance point on the Smith chart reaches the auxiliary circle (two possible solutions). The admittance point will move on the corresponding conductance circle, since the stub does not alter the real part of the admittance

(g) Map the normalized admittance obtained on the auxiliary circle to the location of the second stub d_{stub2}. The point must be on the unitary conductance circle

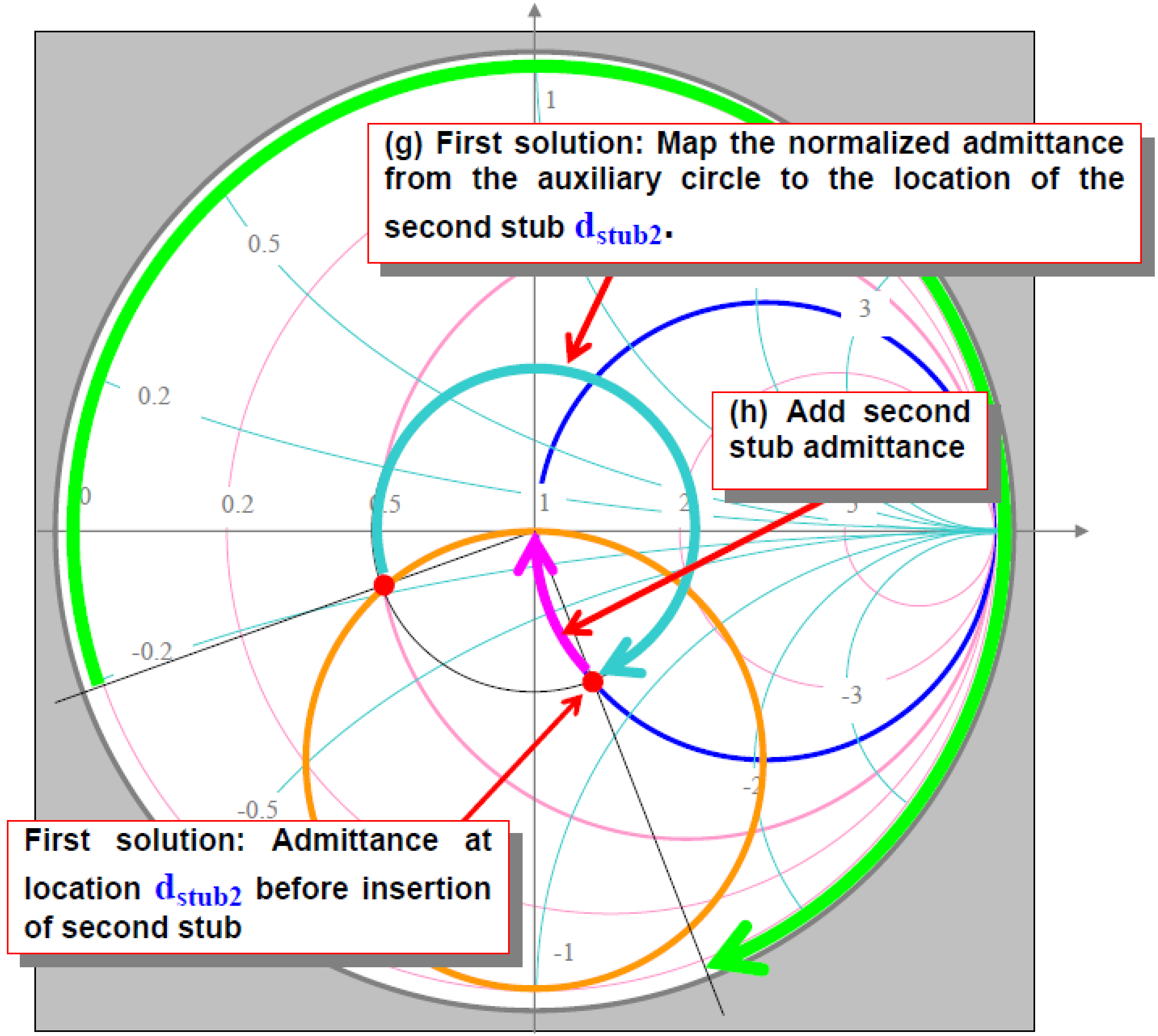
(h) Add the second stub admittance so that the total parallel admittance equals the characteristic admittance of the line to achieve exact matching condition



(f) First solution: Add admittance of first stub to reach auxiliary circle

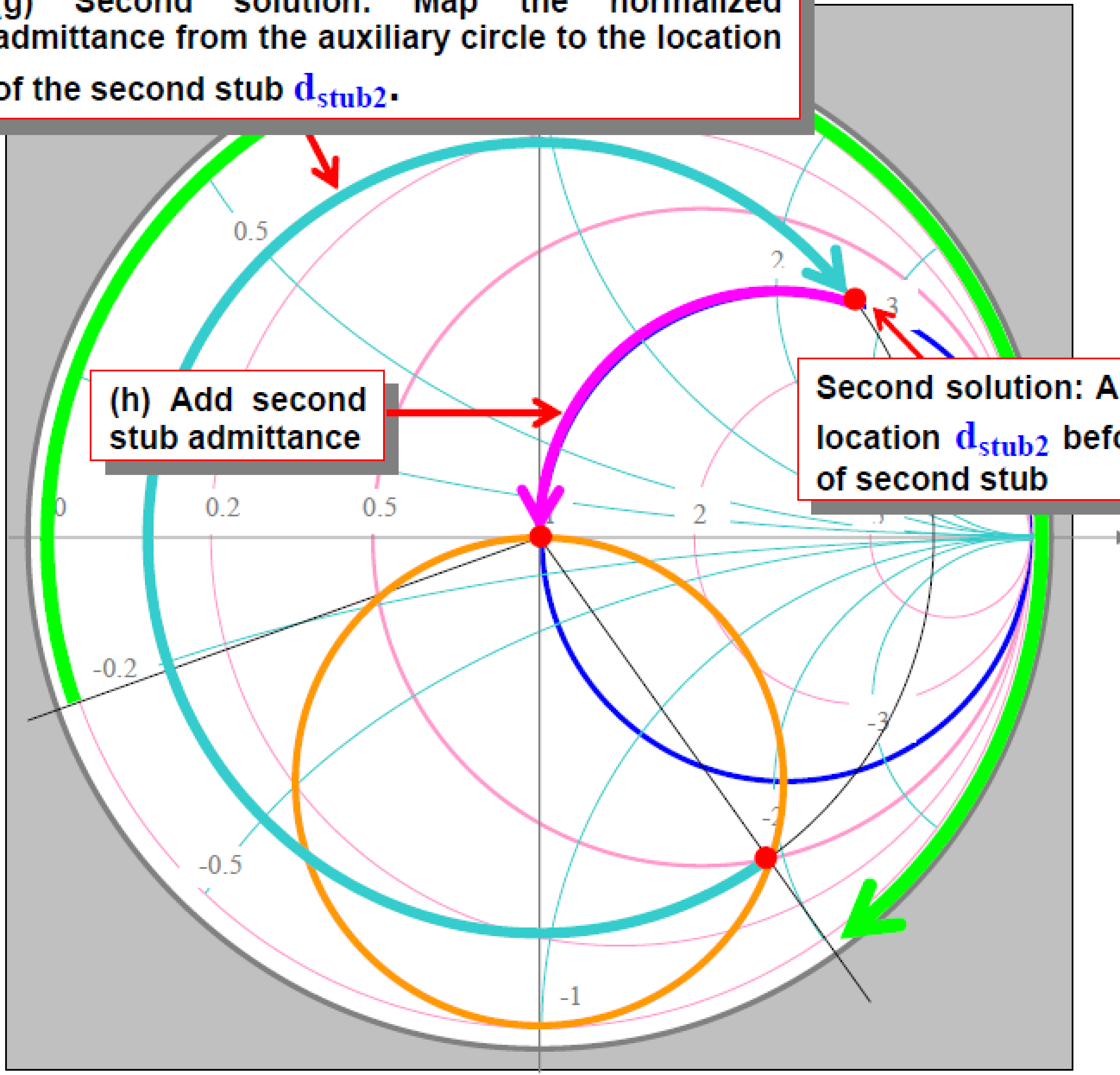
(f) Second solution: Add admittance of first stub to reach auxiliary circle

(e) Draw the auxiliary circle



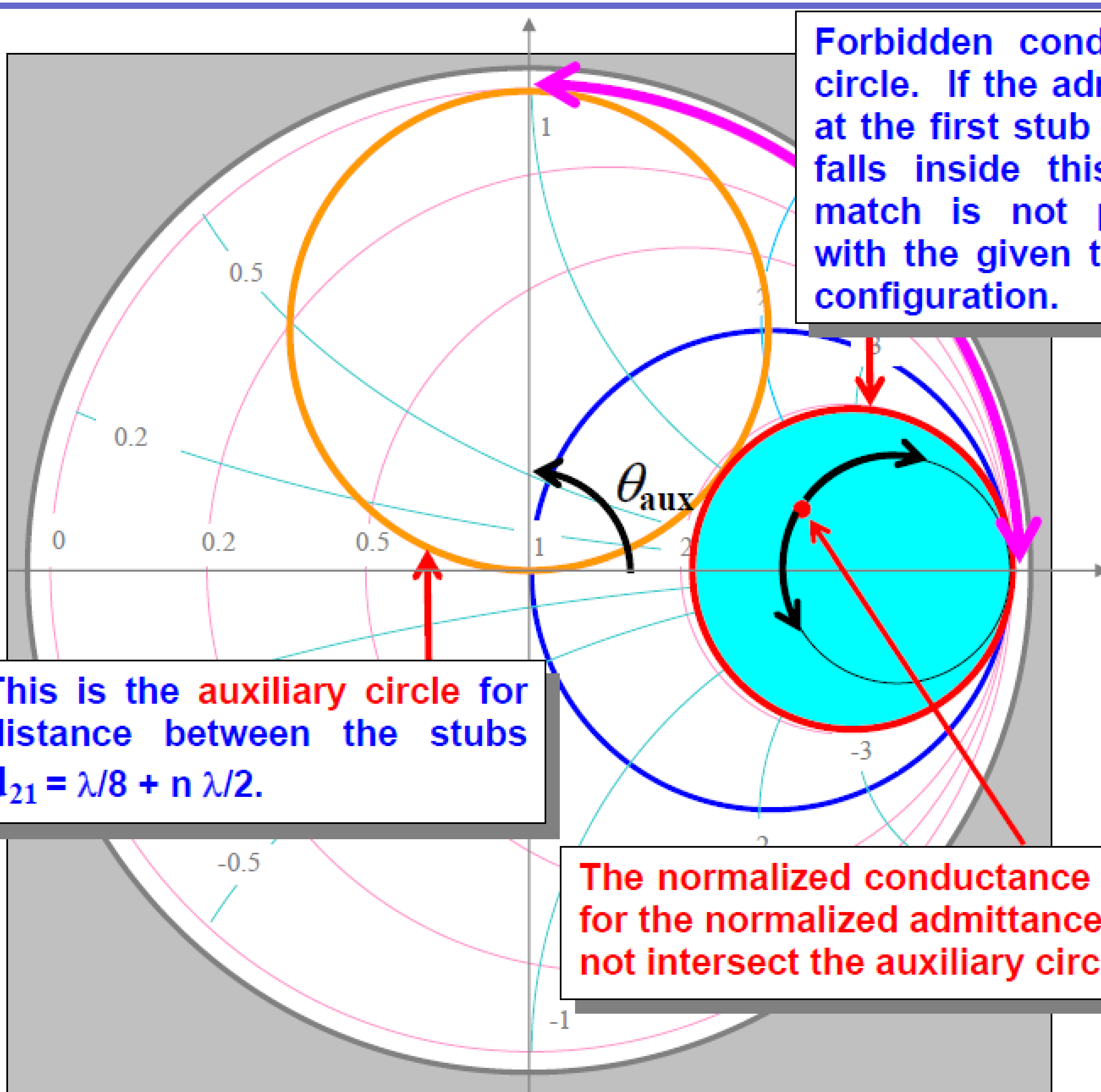
(g) First solution: Map the normalized admittance from the auxiliary circle to the location of the second stub d_{stub2}.
(h) Add second stub admittance
First solution: Admittance at location d_{stub2} before insertion of second stub
1
0.5
0.2
0
0.2
0.5
1
2
3
-0.2
-0.5
-1
-2
-3

(g) Second solution: Map the normalized admittance from the auxiliary circle to the location of the second stub d_{stub2}.



(h) Add second stub admittance

Second solution: Admittance at location d_{stub2} before insertion of second stub

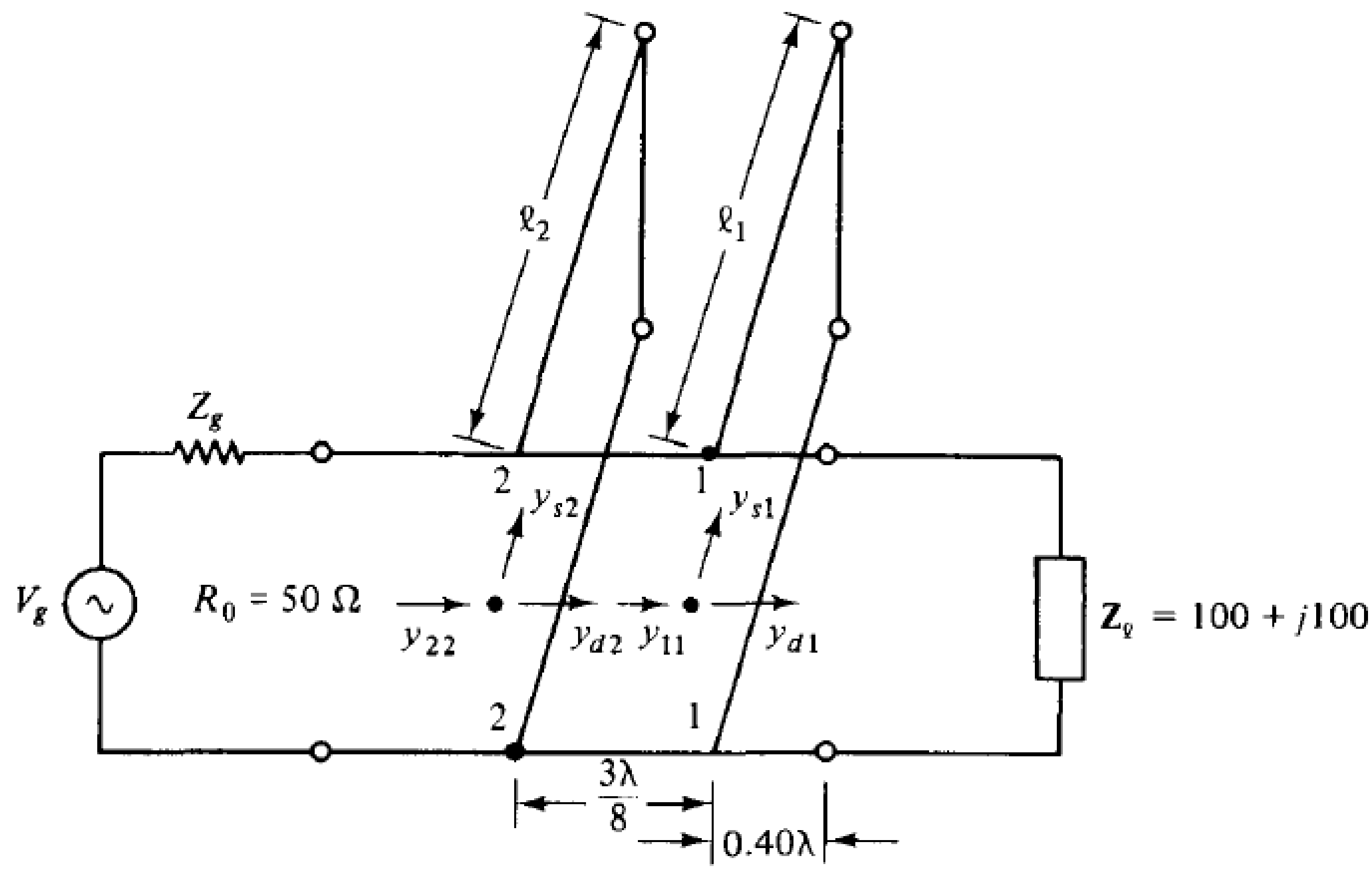


Forbidden conductance circle. If the admittance at the first stub location falls inside this circle, match is not possible with the given two stub configuration.

This is the auxiliary circle for distance between the stubs $d_{21} = \lambda/8 + n\,\lambda/2$.

The normalized conductance circle for the normalized admittance does not intersect the auxiliary circle.

The terminating impedance $\mathbf{Z}_\ell$ is $100 + j100\ \Omega$, and the characteristic impedance $\mathbf{Z}_0$ of the line and stub is $50\ \Omega$. The first stub is placed at 0.40λ away from the load. The spacing between the two stubs is $\frac{3}{8}\lambda$. Determine the length of the short-circuited stubs when the match is achieved. What terminations are forbidden for matching the line by the double-stub device?



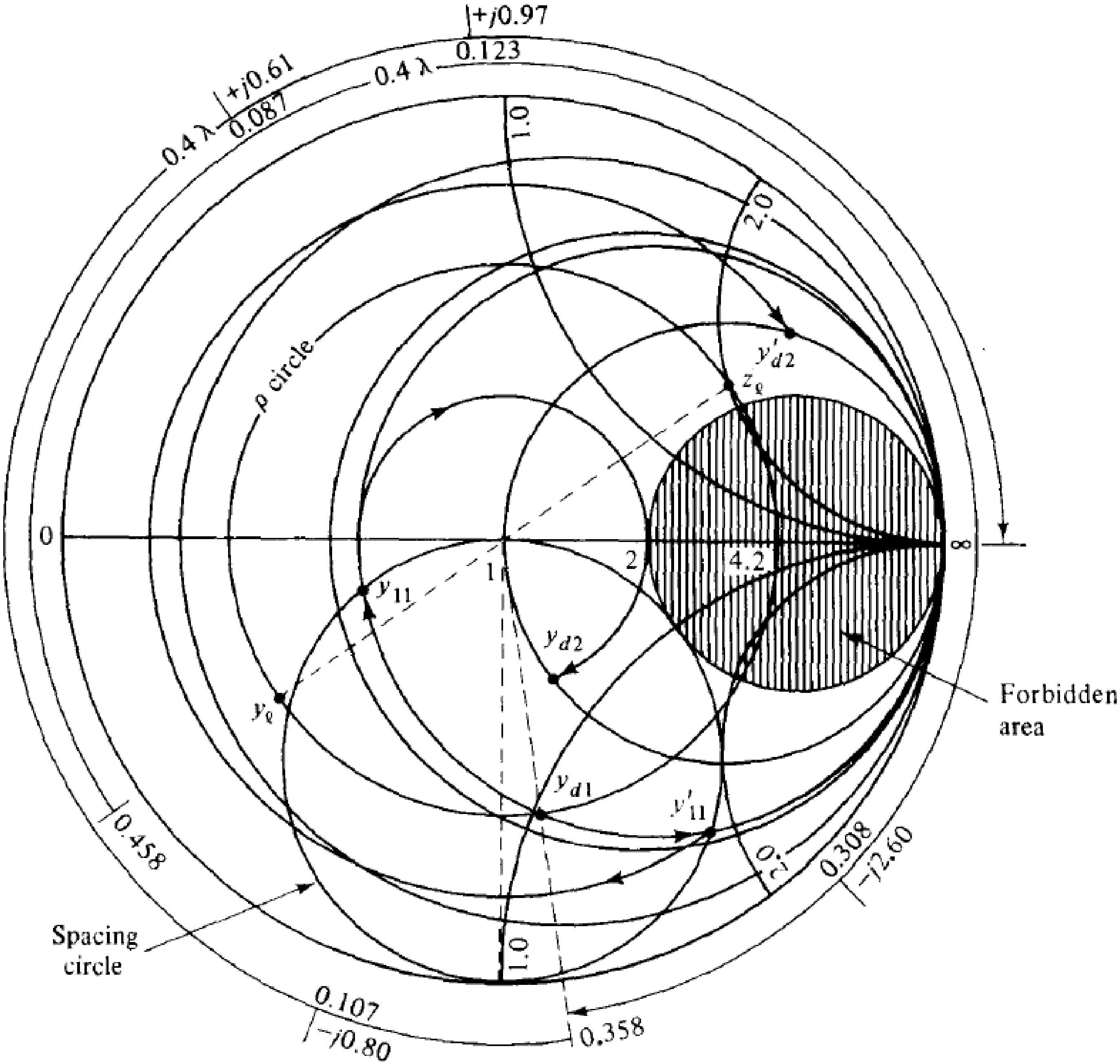
1. Compute the normalized load impedance z_ℓ and enter it on the chart as shown in Fig. 3-6-5:

$$z_\ell = \frac{100 + j100}{50} = 2 + j2$$

2. Plot a SWR ρ circle and read the normalized load admittance 180° out of phase with z_ℓ on the SWR circle:

$$y_\ell = 0.25 - j0.25$$

3. Draw the spacing circle of $\frac{3}{8}\ell$ by rotating the constant-conductance unity circle


+j0.97
0.123
0.4 λ
+j0.61
0.087
0.4 λ
1.0
2.0
ρ circle
y'd2
zℓ
0
2
4.2
∞
1
y11
yd2
yℓ
Forbidden
area
yd1
y'11
0.458
0.308
−j2.60
Spacing
circle
1.0
0.107
−j0.80
0.358

($g = 1$) through a phase angle of $2\beta d = 2\beta\frac{3}{8}\lambda = \frac{3}{2}\pi$ toward the load. Now y_{11} must be on this spacing circle, since y_{d2} will be on the $g = 1$ circle (y_{11} and y_{d2} are $\frac{3}{8}\lambda$ apart).

4. Move y_ℓ for a distance of 0.40λ from 0.458 to 0.358 along the SWR ρ circle toward the generator and read y_{d1} on the chart:

$$y_{d1} = 0.55 - j1.08$$

5. There are two possible solutions for y_{11}. They can be found by carrying y_{d1} along the constant-conductance ($g = 0.55$) circle that intersects the spacing circle at two points:

$$y_{11} = 0.55 - j0.11$$

$$y'_{11} = 0.55 - j1.88$$

6. At the junction 1-1,

$$y_{11} = y_{d1} + y_{s1}$$

Then

$$y_{s1} = y_{11} - y_{d1} = (0.55 - j0.11) - (0.55 - j1.08) = +j0.97$$

Similarly,

$$y'_{s1} = -j.080$$

7. The lengths of stub 1 are found as

$$\ell_1 = (0.25 + 0.123)\lambda = 0.373\lambda$$

$$\ell'_1 = (0.25 - 0.107)\lambda = 0.143\lambda$$

8. The $\frac{3}{8}\lambda$ section of line transforms y_{11} to y_{d2} and y_{11} to y'_{d2} along their constant standing-wave circles, respectively. That is,

$$y_{d2} = 1 - j0.61$$

$$y'_{d2} = 1 + j2.60$$

9. Then stub 2 must contribute

$$y_{s2} = +j0.61$$

$$y'_{s2} = -j2.60$$

10. The lengths of stub 2 are found as

$$\ell_2 = (0.25 + 0.087)\lambda = 0.337\lambda$$

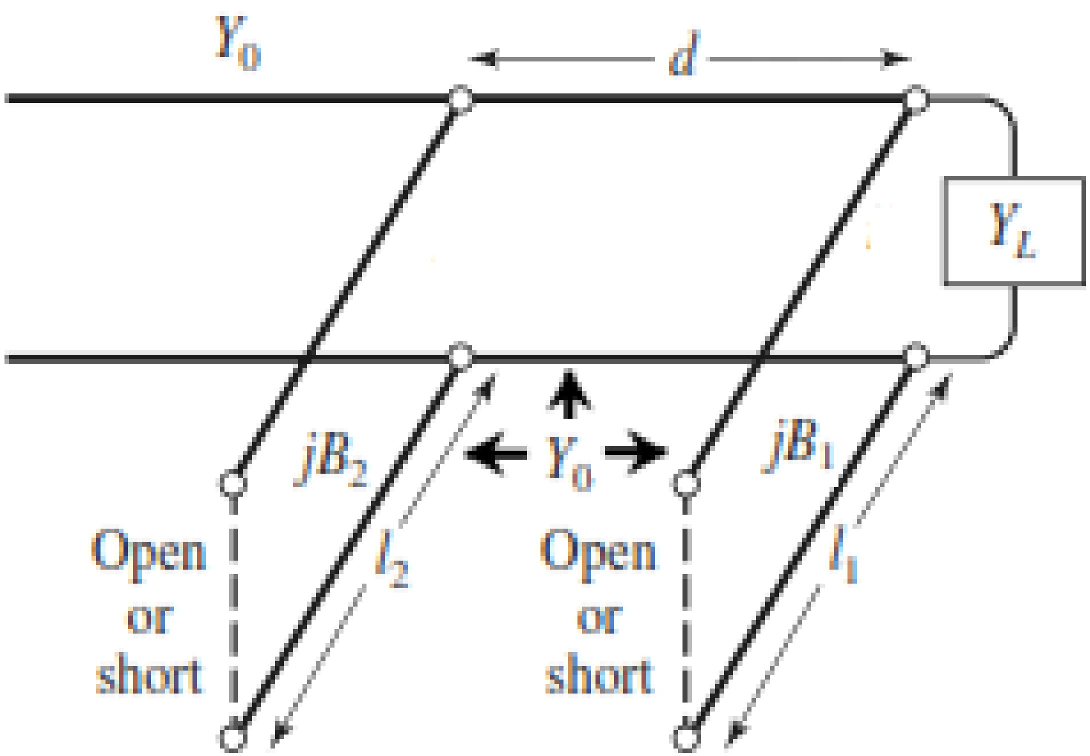
$$\ell'_2 = (0.308 - 0.25)\lambda = 0.058\lambda$$

11. It can be seen from Fig. 3-6-5 that a normalized admittance y_ℓ located inside the hatched area cannot be brought to lie on the locus of y_{11} or y'_{11} for a possible match by the parallel connection of any short-circuited stub because the spacing circle and $g = 2$ circle are mutually tangent. Thus the area of a $g = 2$ circle is called the *forbidden region* of the normalized load admittance for possible match.

Week 8

Slide 149-

DOUBLE STUB MATCHING



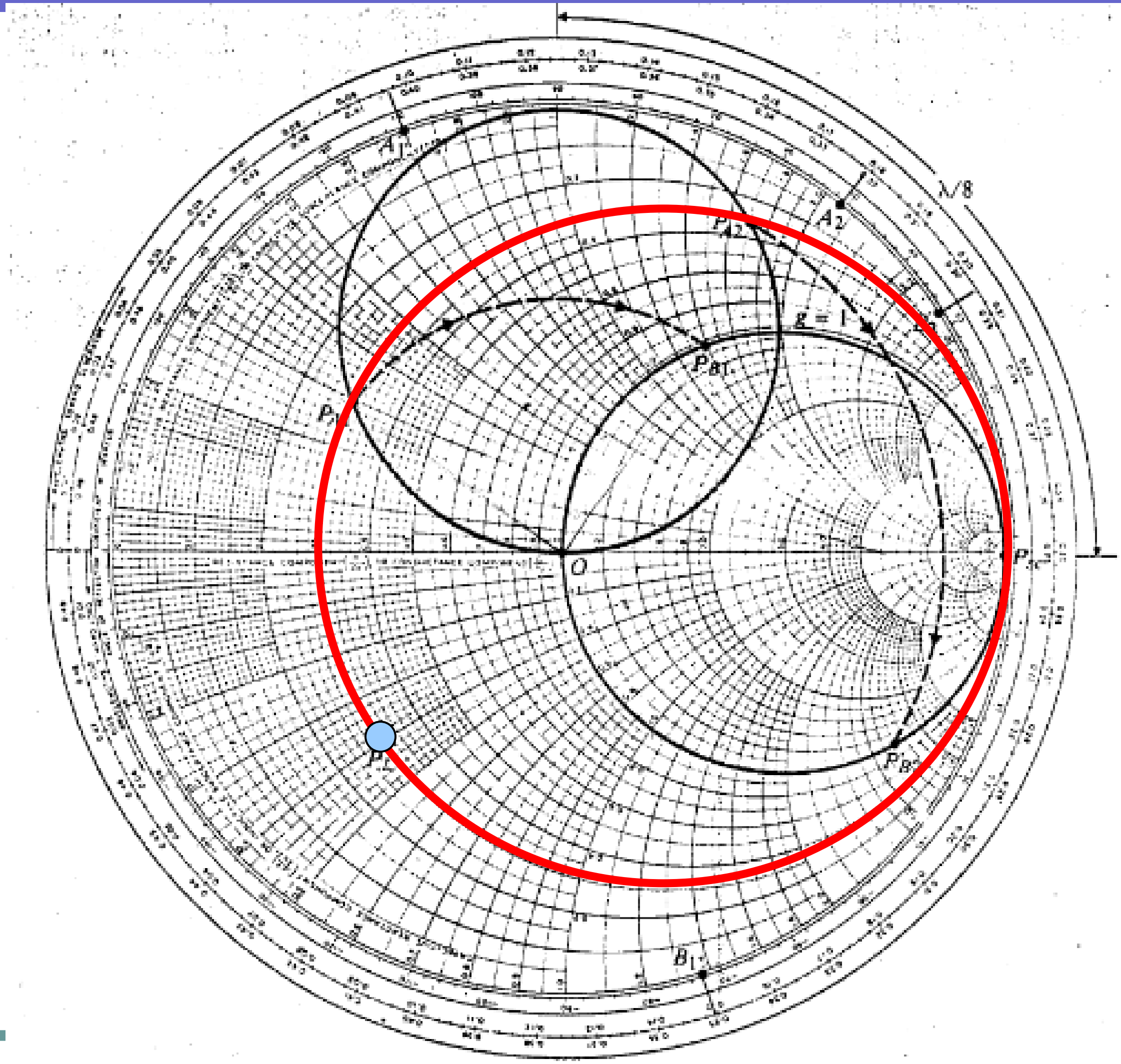
DOUBLE STUB MATCHING

Example 9–17 A 50-(Ω) transmission line is connected to a load impedance $Z_L = 60 + j80$ (Ω). A double-stub tuner spaced an eighth of a wavelength apart is used to match the load to the line, as shown in Fig. 9–24. Find the required lengths of the short-circuited stubs.

Solution: Given $R_0 = 50$ (Ω) and $Z_L = 60 + j80$ (Ω), it is easy to calculate

$$y_L = \frac{1}{z_L} = \frac{R_0}{Z_L} = \frac{50}{60 + j80} = 0.30 - j0.40.$$

DOUBLE STUB MATCHING



DOUBLE STUB MATCHING

1. Draw the $g = 1$ circle (Fig. 9–25).
2. Rotate this $g = 1$ circle by $\frac{1}{8}$ "wavelengths toward load" in the counterclockwise direction. The angle of rotation is $4\pi/8$ (rad) or 90°.
3. Enter $y_L = 0.30 - j0.40$ as P_L.
4. Mark the two points of intersection, P_{A1} and P_{A2}, of the $g_L = 0.30$ circle with the rotated $g = 1$ circle.

$$\text{At } P_{A1}\text{, read } y_{A1} = 0.30 + j0.29;$$
$$\text{At } P_{A2}\text{, read } y_{A2} = 0.30 + j1.75.$$

5. Use a compass centered at the origin O to mark the points P_{B1} and P_{B2} on the $g = 1$ circle corresponding, respectively, to the points P_{A1} and P_{A2}.

$$\text{At } P_{B1}\text{, read } y_{B1} = 1 + j1.38;$$
$$\text{At } P_{B2}\text{, read } y_{B2} = 1 - j3.4.$$

DOUBLE STUB MATCHING

6. Determine the required stub lengths ℓ_{A1} and ℓ_{A2} from

$$(y_{sA})_1 = y_{A1} - y_L = j0.69, \quad \ell_{A1} = (0.097 + 0.250)\lambda = 0.347\lambda \text{ (Point } A_1\text{)},$$
$$(y_{sA})_2 = y_{A2} - y_L = j2.11, \quad \ell_{A2} = (0.179 + 0.250)\lambda = 0.429\lambda \text{ (Point } A_2\text{)}.$$

7. Determine the required stub lengths ℓ_{B1} and ℓ_{B2} from:

$$(y_{sB})_1 = -j1.38, \quad \ell_{B1} = (0.350 - 0.250)\lambda = 0.100\lambda \text{ (Point } B_1\text{)},$$
$$(y_{sB})_2 = j3.4, \quad \ell_{B2} = (0.205 + 0.250)\lambda = 0.455\lambda \text{ (Point } B_2\text{)}.$$

Final Remarks on TL

TL is used to transfer energy from one circuit to another. Moreover

- It can be used as a circuit element such as L/C.
- It can be used as impedance matching device.
- It can be used as stubs.
- To avoid distortion, the relation among the line parameters (R/L=G/C) should be maintained.

EEE 4181

MICROWAVE ENGINEERING

PART 2:

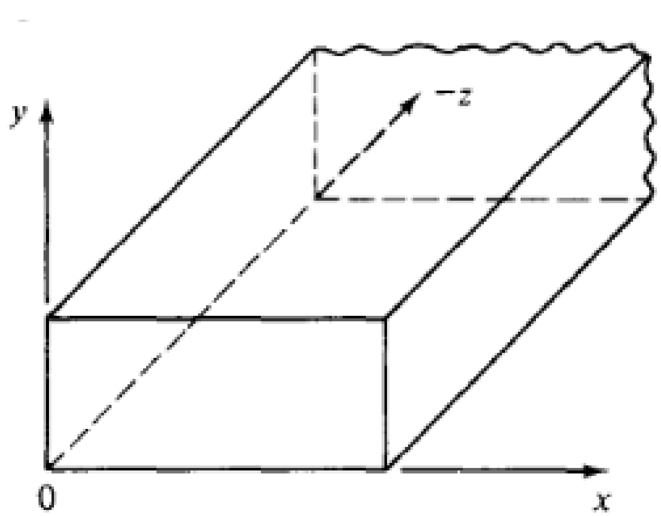
CAVITY RESONATOR

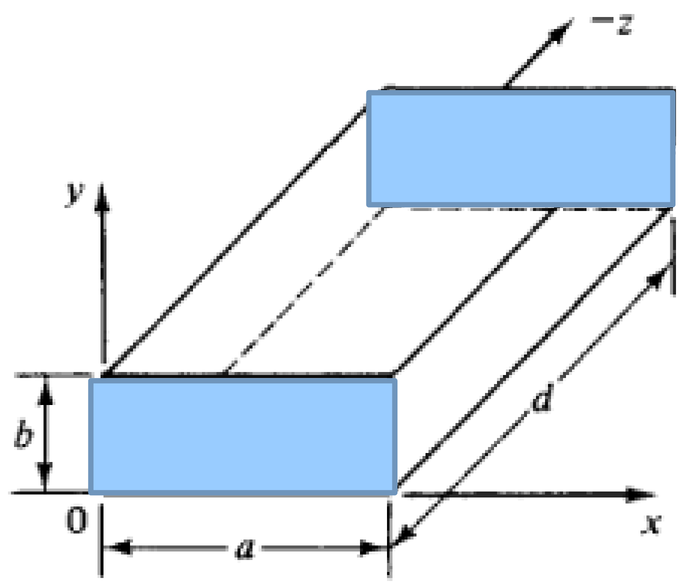
Rectangular Cavity Resonator

- ✓ What is cavity resonator? Where does it use?
- ✓ How does a waveguide cavity work as resonant circuit?
- ✓ Fields inside a rectangular cavity resonator, dominant mode
- ✓ Q factor of cavity resonator.

Rectangular Cavity Resonator

The cavity resonator works as resonant circuit and is obtained from a section of rectangular waveguide closed by two additional highly conducting metal plates.





$$d = p\lambda_g / 2,$$

Rectangular Cavity Resonator

Applications

Cavity resonators are used:

1) as oscillators to generate microwave signals
2) as filters to separate a signal at a given frequency
3) in equipment such as radar, satellite communication, microwave ovens etc
4) To measure the frequency of microwave signals

Rectangular Cavity Resonator

Wave equation:

$$\nabla^2 \overline{E} + \omega^2 \mu\varepsilon \overline{E} = 0$$

Boundary conditions:

$$E_z = 0 \qquad \text{at } x = 0,\ a \quad \text{and } y = 0, b$$

$$\boxed{E_x \propto \frac{\partial E_z}{\partial z} = 0,\ E_y \propto \frac{\partial E_z}{\partial z} = 0} \qquad \text{at } z = 0, d$$

Rectangular Cavity Resonator

TM$_{mnp}$ Wave:

$$E_z = E_{oz} \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)$$

TE$_{mnp}$ Wave:

$$H_z = H_{oz} \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)$$

Rectangular Cavity Resonator

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

Rectangular Cavity Resonator

TM_{mnp} Wave:

$$E_z = E_{oz}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)$$

$$\begin{cases} E_x = -\dfrac{1}{h^2}\left(\dfrac{m\pi}{a}\right)\left(\dfrac{p\pi}{d}\right)E_{0z}\cos\left(\dfrac{m\pi}{a}x\right)\sin\left(\dfrac{n\pi}{b}y\right)\sin\left(\dfrac{p\pi}{d}z\right) \\ E_y = -\dfrac{1}{h^2}\left(\dfrac{n\pi}{b}\right)\left(\dfrac{p\pi}{d}\right)E_{0z}\sin\left(\dfrac{m\pi}{a}x\right)\cos\left(\dfrac{n\pi}{b}y\right)\sin\left(\dfrac{p\pi}{d}z\right) \\ H_x = \dfrac{j\omega\varepsilon}{h^2}\left(\dfrac{n\pi}{b}\right)E_{0z}\sin\left(\dfrac{m\pi}{a}x\right)\cos\left(\dfrac{n\pi}{b}y\right)\cos\left(\dfrac{p\pi}{d}z\right) \\ H_y = -\dfrac{j\omega\varepsilon}{h^2}\left(\dfrac{m\pi}{a}\right)E_{0z}\cos\left(\dfrac{m\pi}{a}x\right)\sin\left(\dfrac{n\pi}{b}y\right)\cos\left(\dfrac{p\pi}{d}z\right) \end{cases}$$

Rectangular Cavity Resonator

TE_{mnp} Wave:

$$H_z = H_{oz}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)$$

$$\begin{cases}
E_x = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right)H_{0z}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \\
E_y = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_{0z}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \\
H_x = -\frac{1}{h^2}\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)H_{0z}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \\
H_y = -\frac{1}{h^2}\left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)H_{0z}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right)
\end{cases}$$

Rectangular Cavity Resonator

$$h^2 = \omega^2 \mu\varepsilon - \beta^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

m, n, and p are the number of half cycle of the waves along x, y, and z axes.

For TM wave neither *m* nor *n* =0, but *p* can be 0.

For TE wave *p*≠0, either *m* or *n* =0, but not both.

Rectangular Cavity Resonator

The of new set of conducting plates introduce a condition for standing waves in the z-direction which leads to the following oscillation frequencies.

$$f_r = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2}$$

$$\lambda_r = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2}}.$$

The high-pass behavior of the rectangular wave guide is modified into a <u>very narrow</u> pass-band behavior, since cut-off frequencies of the wave guide are transformed into oscillation frequencies of the resonator.

Rectangular Cavity Resonator

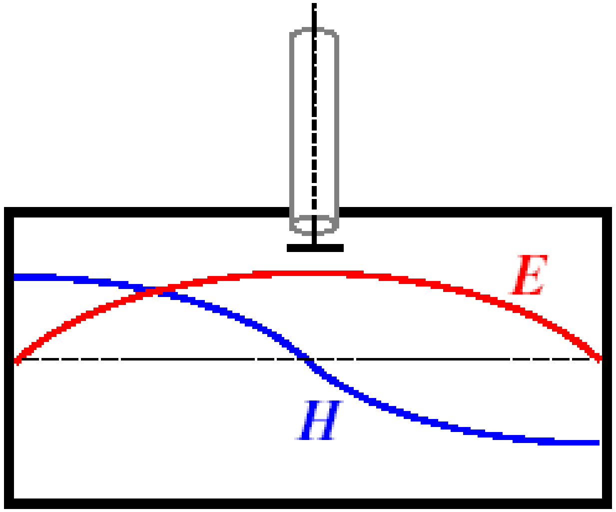
The wave with lowest resonance frequency is called the dominant mode and the waves with same resonance frequency are called degenerative modes.

For d>a>b, the dominant mode is TE_{101}.

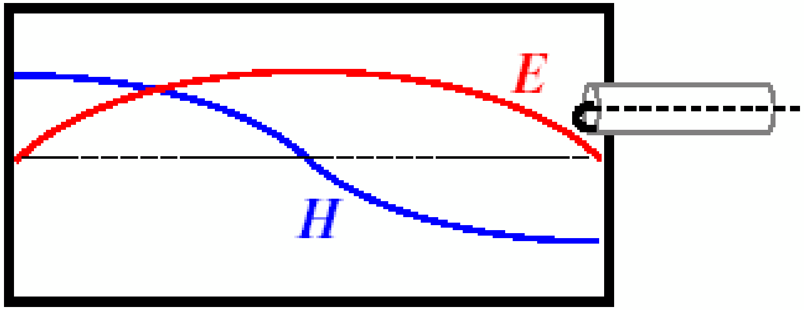
For a>b>d, the dominant mode is TM_{110}.

Rectangular Cavity Resonator

Excitation method:



Excitation with a dipole antenna



Excitation with a loop antenna

Rectangular Cavity Resonator

The quality factor is in general a measure of the ability of a resonator to store energy in relation to time-average power dissipation. Specifically, the Q of a resonator is defined as

$$Q = 2\pi \frac{maximum\ energy\ stored}{energy\ dissipated\ per\ cycle} = \frac{\omega W}{P}$$

$W = W_e + W_m$

Note that the the time-average electric and magnetic energies are precisely equal. Physically, over a period, it is shared equally between the electric and magnetic forms.

Rectangular Cavity Resonator

For dominant mode TE_{101}

$$\begin{cases} H_z = H_{oz}\cos\left(\dfrac{\pi}{a}x\right)\sin\left(\dfrac{\pi}{d}z\right) \\ E_y = -\dfrac{j\omega\mu}{h^2}(\dfrac{\pi}{a})H_{0z}\sin(\dfrac{\pi}{a}x)\sin(\dfrac{\pi}{d}z) \\ H_x = -\dfrac{1}{h^2}(\dfrac{\pi}{a})(\dfrac{\pi}{d})H_{0z}\sin(\dfrac{\pi}{a}x)\cos(\dfrac{\pi}{d}z) \end{cases}$$

$$W_e = \frac{\varepsilon}{4}\int\left|E_y\right|^2 dv \qquad W_m = \frac{\mu}{4}\int\{\left|H_x\right|^2 + \left|H_y\right|^2\}dv$$

Rectangular Cavity Resonator

At resonance

$$W = \frac{\mu}{8} \cdot H_{0z}^{2} abd\left(\frac{a^{2}}{d^{2}} + 1\right)$$

$$P = \frac{R_s H_{0z}^{2} d^{2}}{4}\left\{\frac{a}{d}\left(\frac{a^{2}}{d^{2}} + 1\right) + \frac{2b}{d}\left(\frac{a^{3}}{d^{3}} + 1\right)\right\}$$

$$\Rightarrow Q_{TE_{101}} = \frac{\pi f_{101} \mu abd(a^{2} + d^{2})}{R_s[2b(a^{3} + d^{3}) + ad(a^{2} + d^{2})]}$$

$$Q_{TM_{110}} = \frac{\pi (f_r)_{TM_{110}} \mu_0 abd(a^{2} + b^{2})}{R_s[2d(a^{3} + d^{3}) + ab(a^{2} + b^{2})]}$$

Rectangular Cavity Resonator

EXAMPLE

An air-filled resonant cavity with dimensions $a = 5$ cm, $b = 4$ cm, and $c = 10$ cm is made of copper ($\sigma_c = 5.8 \times 10^7$ mhos/m). Find

(a) The five lowest order modes

(b) The quality factor for TE_{101} mode

Answer:

Thus the five lowest order modes in ascending order are

TE_{101}	(3.35 GHz)
TE_{011}	(4.04 GHz)
TE_{102}	(4.243 GHz)
TM_{110}	(4.8 GHz)
TE_{111} or TM_{111}	(5.031 GHz)

$$Q_{TE_{101}} = 14{,}358$$

Rectangular Cavity Resonator

Example 10–11 Determine the dominant modes and their frequencies in an air-filled rectangular cavity resonator for (a) $a > b > d$, (b) $a > d > b$, and (c) $a = b = d$, where a, b, and d are the dimensions in the x, y, and z directions respectively.

Answer: (a) TM_{110} (b) TE_{101} (c) TM_{110} TE_{101} TE_{011} degenerate mode

Example 10–12 (a) What should be the size of a hollow cubic cavity made of copper in order for it to have a dominant resonant frequency of 10 (GHz)? (b) Find the Q at that frequency.

Answer: (a) TM_{110} TE_{101} TE_{011} degenerate mode, a=21.2 cm (b) 10700.

Week 9

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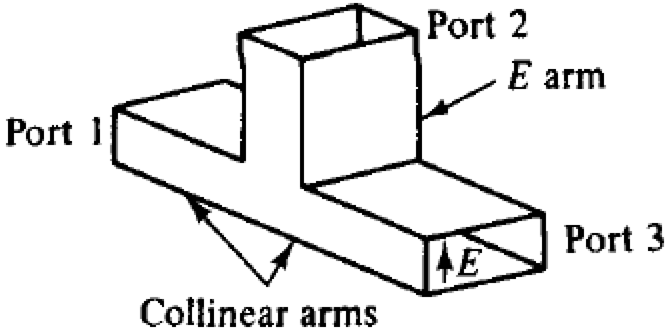
EEE 4181

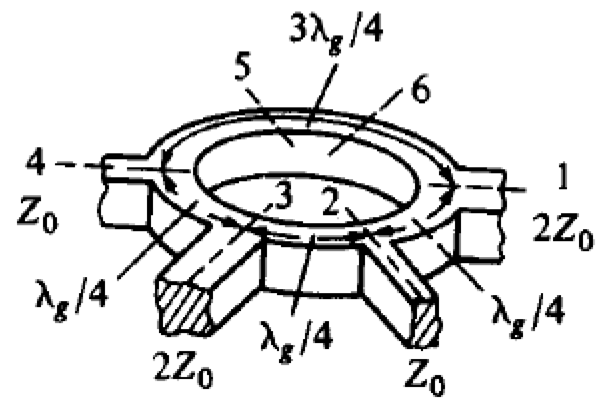
MICROWAVE ENGINEERING

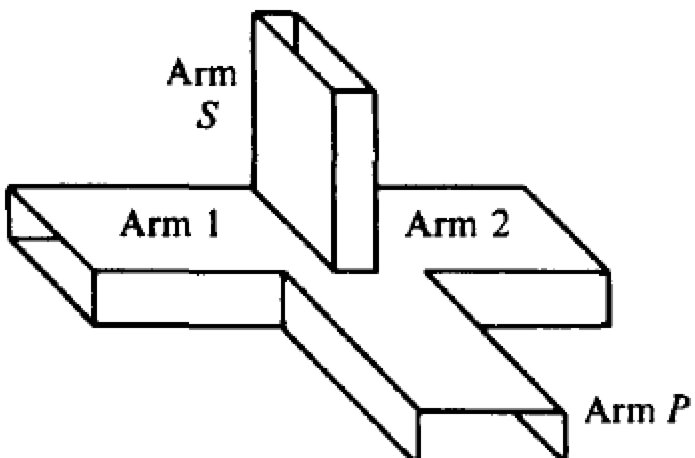
PART 3:

S-parameters & Microwave Junctions

Microwave Junctions







Why S-parameters?

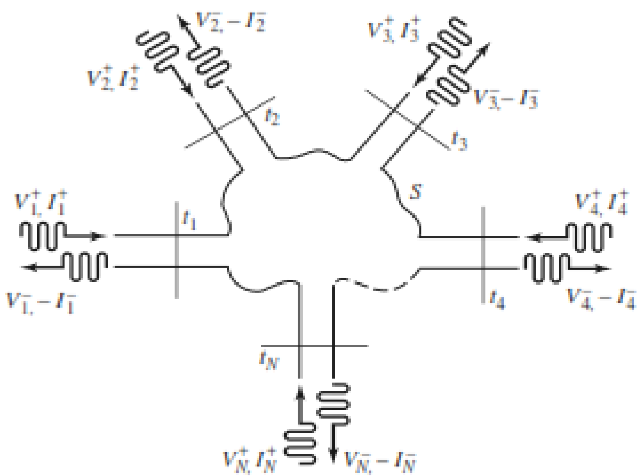
At low frequency, Networks are analyzed by Z, Y, h, and ABCD parameters.

$$\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{21} & . & . & Z_{n1} \\ Z_{12} & Z_{22} & . & . & Z_{n2} \\ . & . & . & . & . \\ . & . & . & . & . \\ Z_{1n} & Z_{2n} & . & . & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{21} & . & . & Y_{n1} \\ Y_{12} & Y_{22} & . & . & Y_{n2} \\ . & . & . & . & . \\ . & . & . & . & . \\ Y_{1n} & Y_{2n} & . & . & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$

At high frequency, Z, Y, h, and ABCD parameters do not work, this is simply because we can not create perfect opens or shorts which is required to determine the different elements.

What is S-parameters?

S-parameters describe a given network in terms of waves rather than voltages or currents.



$$\begin{bmatrix} V_1^- \\ V_2^- \\ \cdot \\ \cdot \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot & S_{1N} \\ S_{21} & S_{22} & \cdot & \cdot & S_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{N1} & \cdot & \cdot & \cdot & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \cdot \\ \cdot \\ V_N^+ \end{bmatrix}$$

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

Elements of S-matrix

S_{11}, S_{22}, S_{33}, …… S_{NN} are the input reflection coefficients when all other ports are matched terminated. Thus S_{NN} may be determined by connecting a generator to port N and matched loads to other ports. Since, there is no power input in other ports except port N:

$$S_{NN} = \frac{V_N^-}{V_N^+}$$

The off-diagonal elements represent transmission coefficients.

$S_{ij}=V^-_i/V^+_j$ where V^-_i is the wave that comes out from port i when a generator of wave V^+_j is connected to port j and all other ports are terminated to matched loads.

Properties of S-parameters

1. NxN matrix.
2. If all the ports are perfectly matched, then the leading diagonal elements will all be zero, i. e., $S_{ii}=0$.
3. For any lossless linear and reciprocal network, in absence of active devices, the S-parameters are equal to their correspon- ding transposes, i. e., $[S]=[S]^t$ Symmetric Network
4. For any lossless network, the sum of the products of each term of any column (or row) of the [S] matrix with its complex conjugate is unity

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 1 \qquad \text{for } i = j$$

5. The sum of the products of each term of any column (or row) of the [S] matrix with the complex conjugate of the corresponding term of a different column (or row) is zero

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0 \qquad \text{for } i \neq j$$

Example

A certain two-port network is measured and the following scattering matrix is obtained:

$$[S] = \begin{bmatrix} 0.1\angle 0^o & 0.8\angle 90^o \\ 0.8\angle 90^o & 0.2\angle 0^o \end{bmatrix}$$

From the data , determine whether the network is reciprocal or lossless. If a short circuit is placed on port 2, what will be the resulting return loss at port 1?

<u>Solution</u>

Since [S] has symmetry, the network is reciprocal. To be lossless, the S parameters must satisfy

$$\sum_{k=1}^{n} S_{ki} S_{kj}^{*} = \begin{cases} 1 & for\, i = j \\ 0 & for\, i \neq j \end{cases}$$

$|S_{11}|^2 + |S_{21}|2 = (0.1)^2 + (0.8)^2 = 0.65$

Since the summation is not equal to 1, thus it is not a lossless network.

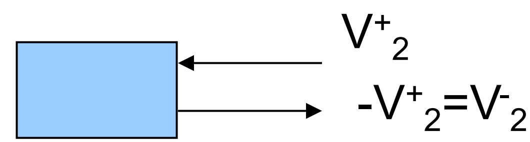
continue

Reflected power at port 1 when port 2 is shorted can be calculated as follow and the fact that $V^+_2 = -V^-_2$ for $\Gamma=-1$ at port 2.

$$V^-_1 = S_{11}V^+_1 + S_{12}V^+_2 = S_{11}V^+_1 - S_{12}V^-_2 \quad (1)$$

$$V^-_2 = S_{21}V^+_1 + S_{22}V^+_2 = S_{21}V^+_1 - S_{22}V^-_2 \quad (2)$$

Short at port 2



From (2) we have

$$V_2^- = \frac{S_{21}}{1+S_{22}} V_1^+ \qquad (3)$$

Dividing (1) by V^+_1 and substitute the result in (3) ,we have

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12}\frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}} = 0.1 - \frac{(j0.8)(j0.8)}{1+0.2} = 0.633$$

Return loss $\equiv 20\log\Gamma = -20\log(0.633) = 3.97\,dB$

Waveguide Tees: General Properties

Waveguide Tees are 3-port Networks.

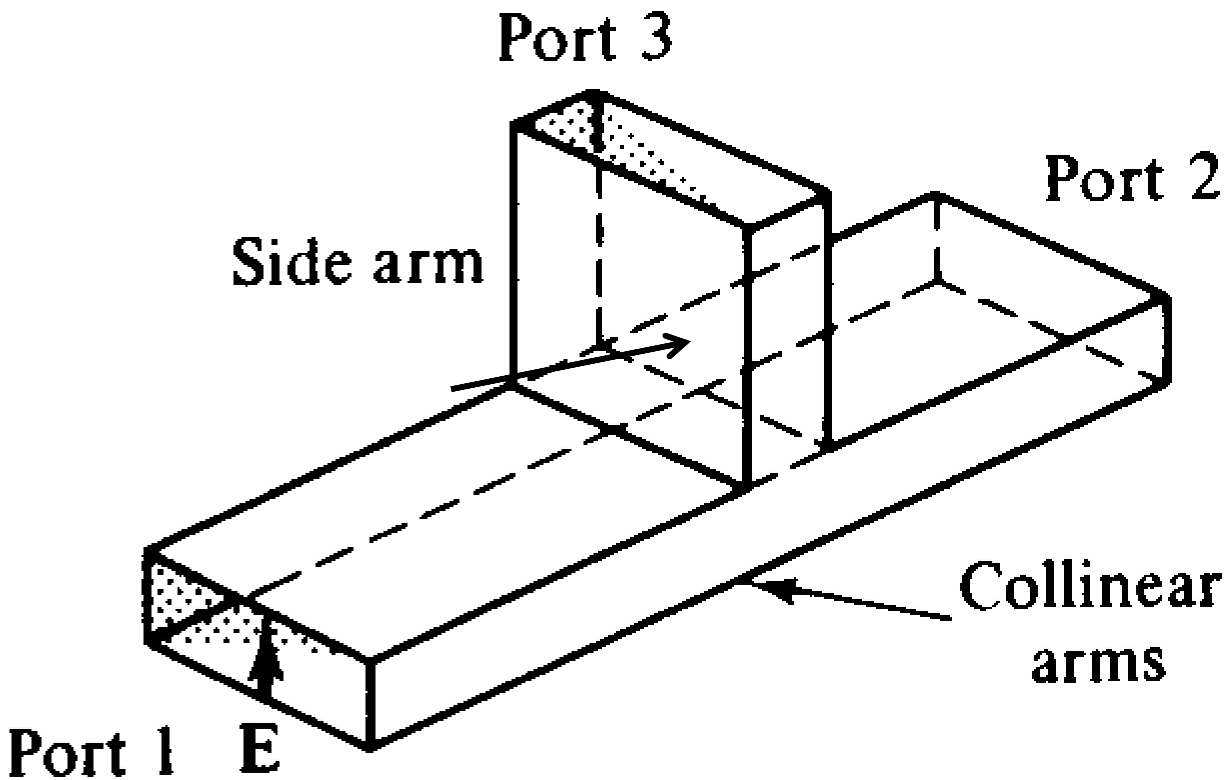
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

A three-port network cannot be simultaneously lossless, reciprocal, and matched at all ports.

Waveguide Tees

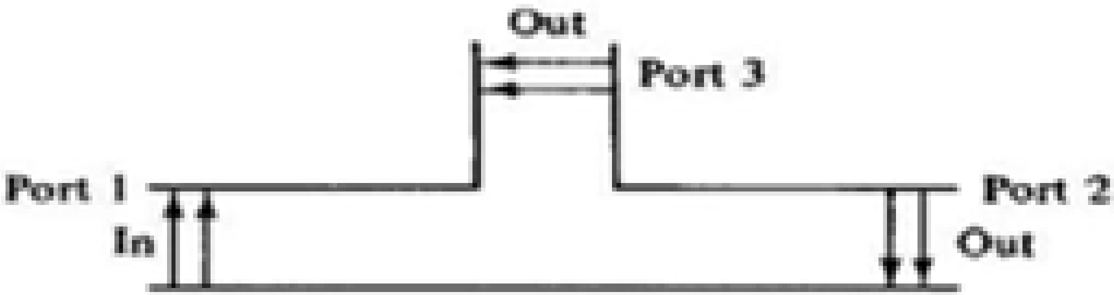
Waveguide Tees are used to branch the main waveguide/transmission line.

✓ E-plane Tee: E-field is parallel to the axis of the side arm

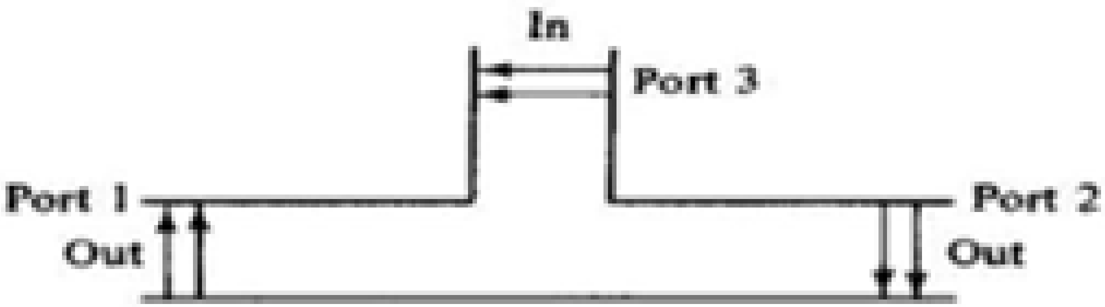


E-plane Tee

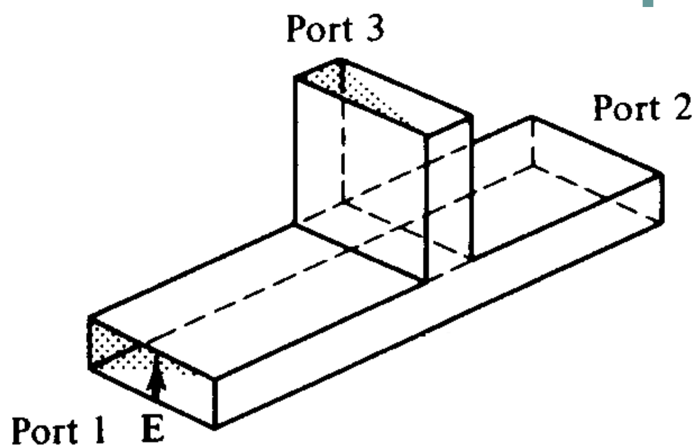
Two different transmission characteristics are:



(a)



(b)



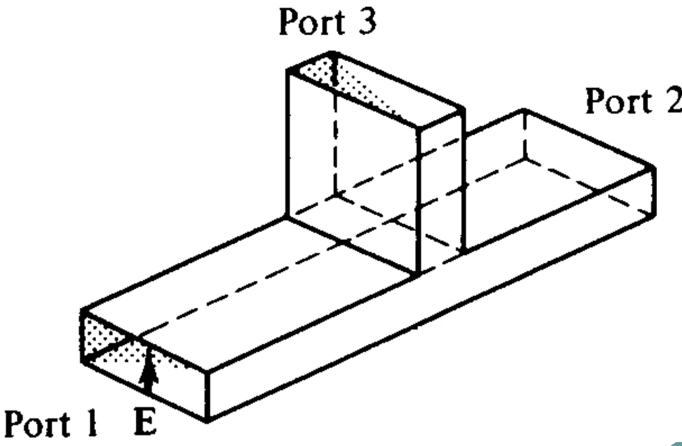
E-plane Tee

(With port 3 is matched)

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

✓An input at port 3 is equally divided between port 1 and port 2 but introduces a phase shift of 180^0 between the two outputs.

✓Equal inputs at port 1 and por results in no output at port 3.

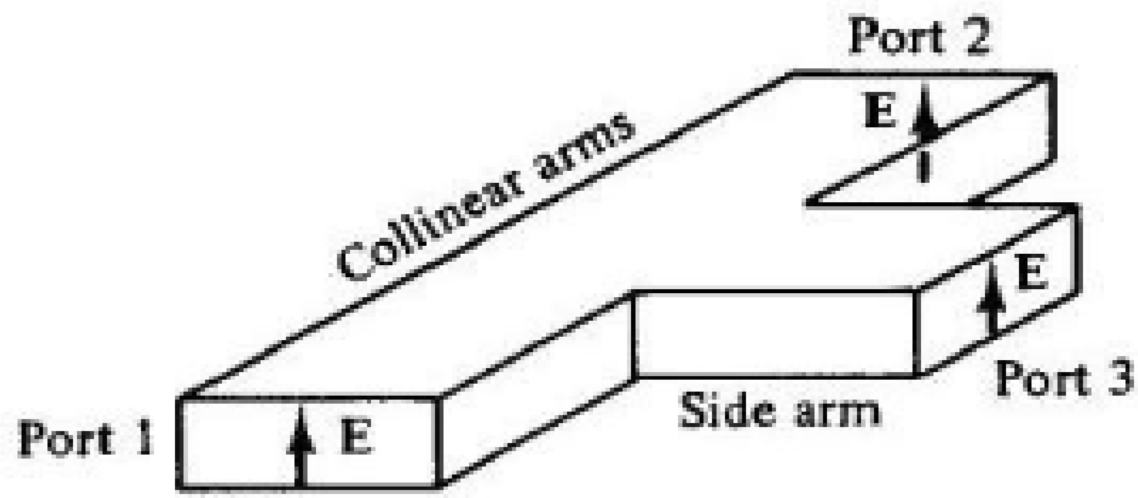


In a E-plane tee junction, 20 mW power is applied to port 3 that is perfectly matched. Calculate the power delivered to the load 60Ω and 75Ω, connected to ports 1 and 2, respectively. Assume the characteristic impedance of the collinear arm of the tee is 50Ω.

$$\Gamma_1 = \frac{60-50}{60+50} = 0.091 \quad \text{and} \quad \Gamma_2 = \frac{75-50}{75+50} = 0.2$$

The actual power delivered to the load at port 1 is $10(1-0.091^2)$mW=9.92mW and that at port 2 is $10(1-0.2^2)$mW =9.6mW

H-plane Tee



Axis of its side arm is parallel to the H-field of the main guide.

H-plane Tee

$$S = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

(With port 3 is matched)

✓An input at port 3 is equally divided between port 1 and port 2.

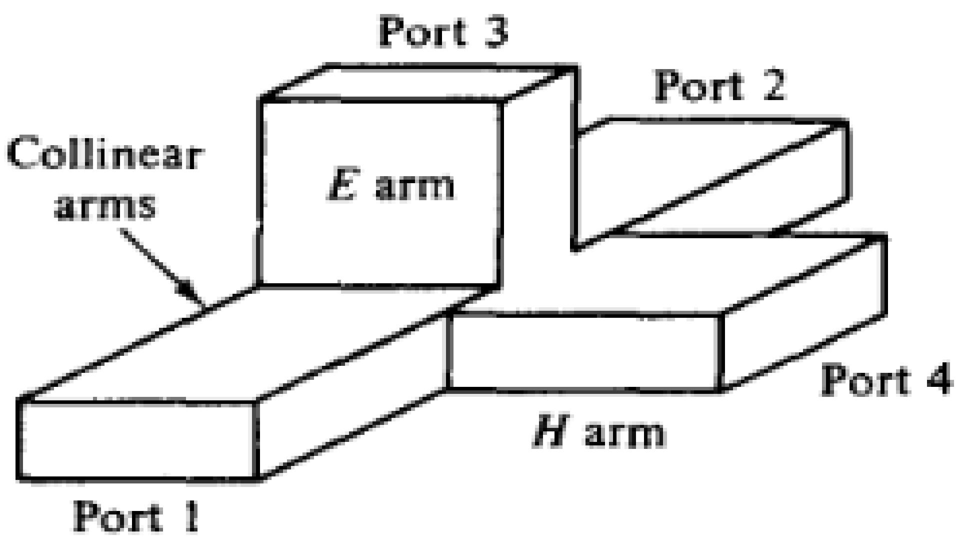
✓If port 1 and port 2 have equal inputs, the output wave at port 3 will be in phase and additive.

Week 10

Slide 190-204

Magic Tee

Combination of the E-plane tee and H-plane tee. It is commonly used for mixing, duplexing, and impedance matching.



Magic Tee

Characteristics:

✓If two waves of equal magnitude and same phase are fed into port 1 and port 2, output will be zero at port 3 and additive at port 4.

✓If a wave is fed into port 4 (the H arm), it will be divide equally between port 1 and port 2 of the collinear arm and will not appear at port 3 (the E arm).

✓If a wave is fed into port 3 (the E arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. The output at port 4 is zero (the H arm).

✓If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1.

Magic Tee

From (2), we have $S_{14}=S_{24}$, $S_{34}=0$.

From (3), $S_{13}=-S_{23}$ and $S_{43}=0$.

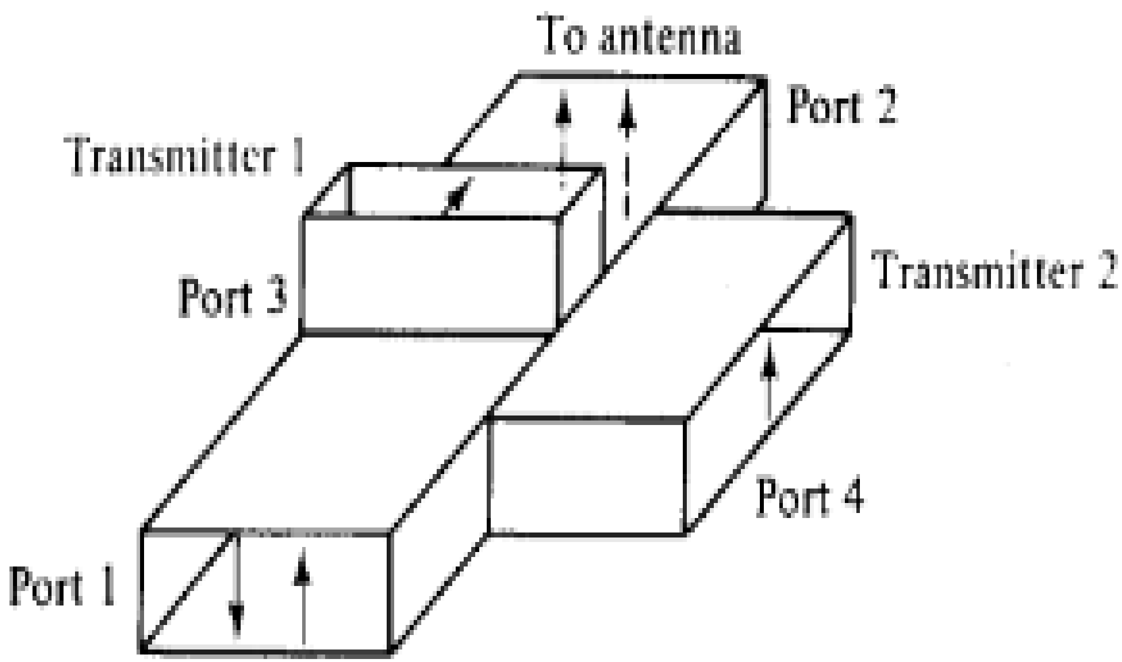
From (4), $S_{21}=S_{12}=0$.

Again, from the symmetry $S_{12}=S_{21}$, $S_{13}=S_{31}$, $S_{14}=S_{41}$, $S_{34}=S_{43}$, $S_{23}=S_{32}$, and $S_{24}=S_{42}$.

And from the matching conditions at port 3 and port 4, $S_{33}=S_{44}=0$.

$$S = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

Magic Tee



See Samuel Y. Liao

Major Applications of Magic Tee

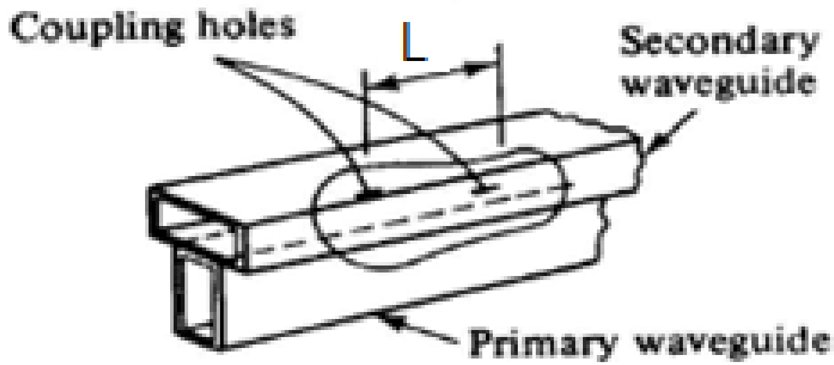
E-H Plane Tee/Magic Tee is used as a duplexer

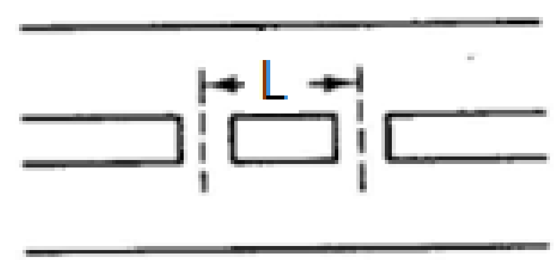
E-H Plane Tee is used as a mixer

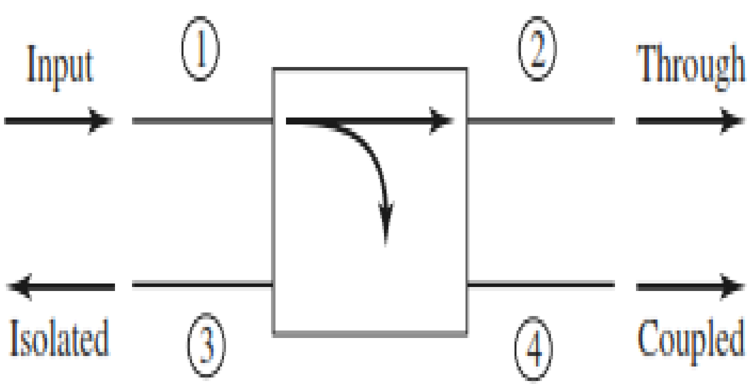
The E-H plane Tee or magic Tee may also be used to couple
two transmitters to an antenna to enhance the input power of the antenna.

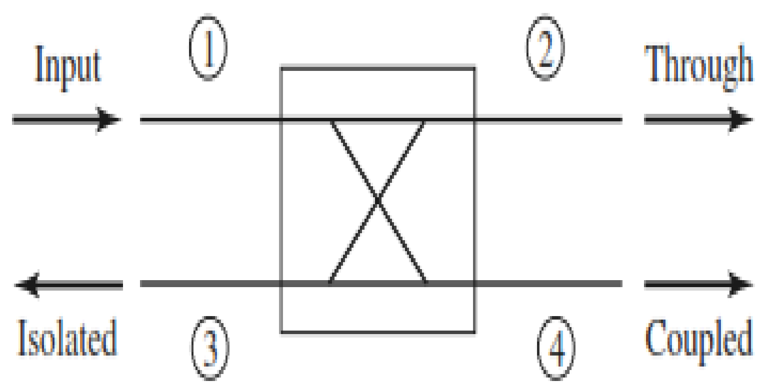
Directional Coupler

Directional couplers are 4-port passive devices used for power division.







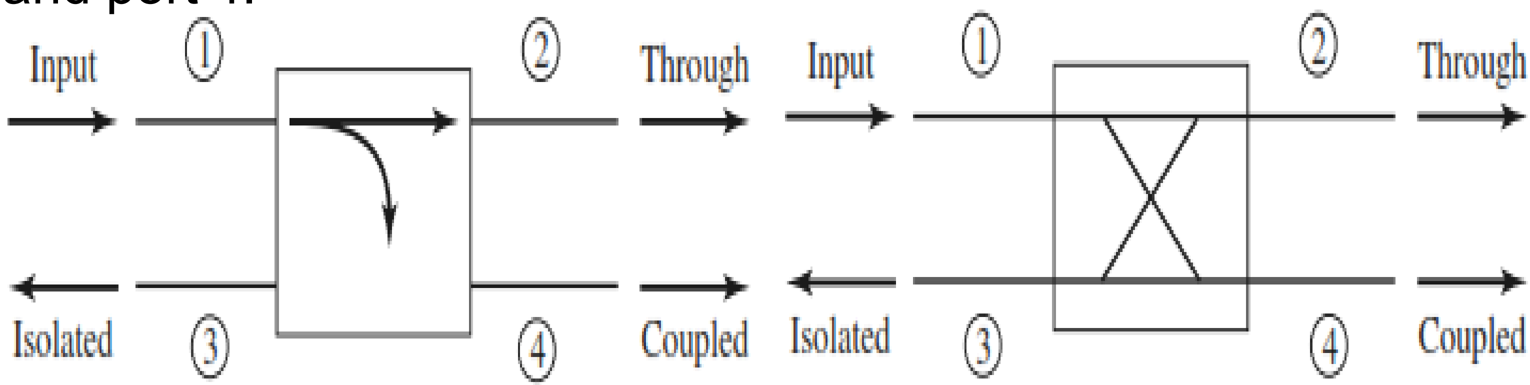


Directional Coupler

Directional Coupler

The properties of an ideal directional coupler:

1) A portion of power traveling from port 1 to port 2 is coupled to port 4 but not to port 3.
2) A portion of power traveling from port 2 to port 1 is coupled to port 3 but not to port 4 (bi-directional case).
3) A portion of power incident on port 3 is coupled to port 2 but not to port 1 and a portion of power incident on port 4 is coupled to port 1 but not to port 2. Also port 1 and port 3 are decoupled as are port 2 and port 4.



Directional Coupler

$$Coupling\ factor = 10\log_{10}\frac{P_1}{P_4} = \text{-}20\log_{10}\left|S_{14}\right|\ (dB)$$

$$Directivity = 10\log_{10}\frac{P_4}{P_3} = \text{-}20\log_{10}\left|\frac{S_{14}}{S_{13}}\right|(dB)$$

$$Isolation(dB) = 10\ log_{10}\frac{P_1}{P_3} = -20\ log_{10}|S_{13}|$$

$$= Coupling\ factor + Directivity.$$

P_1=power input to port 1
P_3=power output from port 3
P_4=power forwarded from input port (port 1) to the coupled port (port 4)

Directional Coupler

With the condition $S_{11}=S_{22}=S_{33}=S_{44}=0$ and the properties $S_{13}=S_{31}=S_{24}=S_{42}=0$.

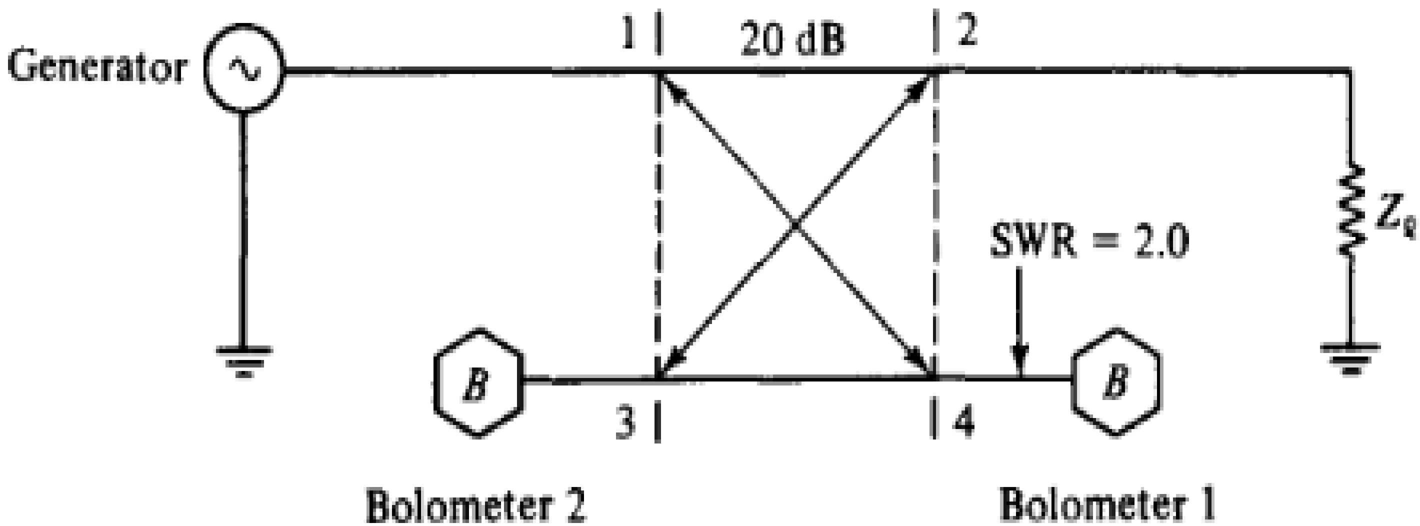
$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

$$p^2 + q^2 = 1$$

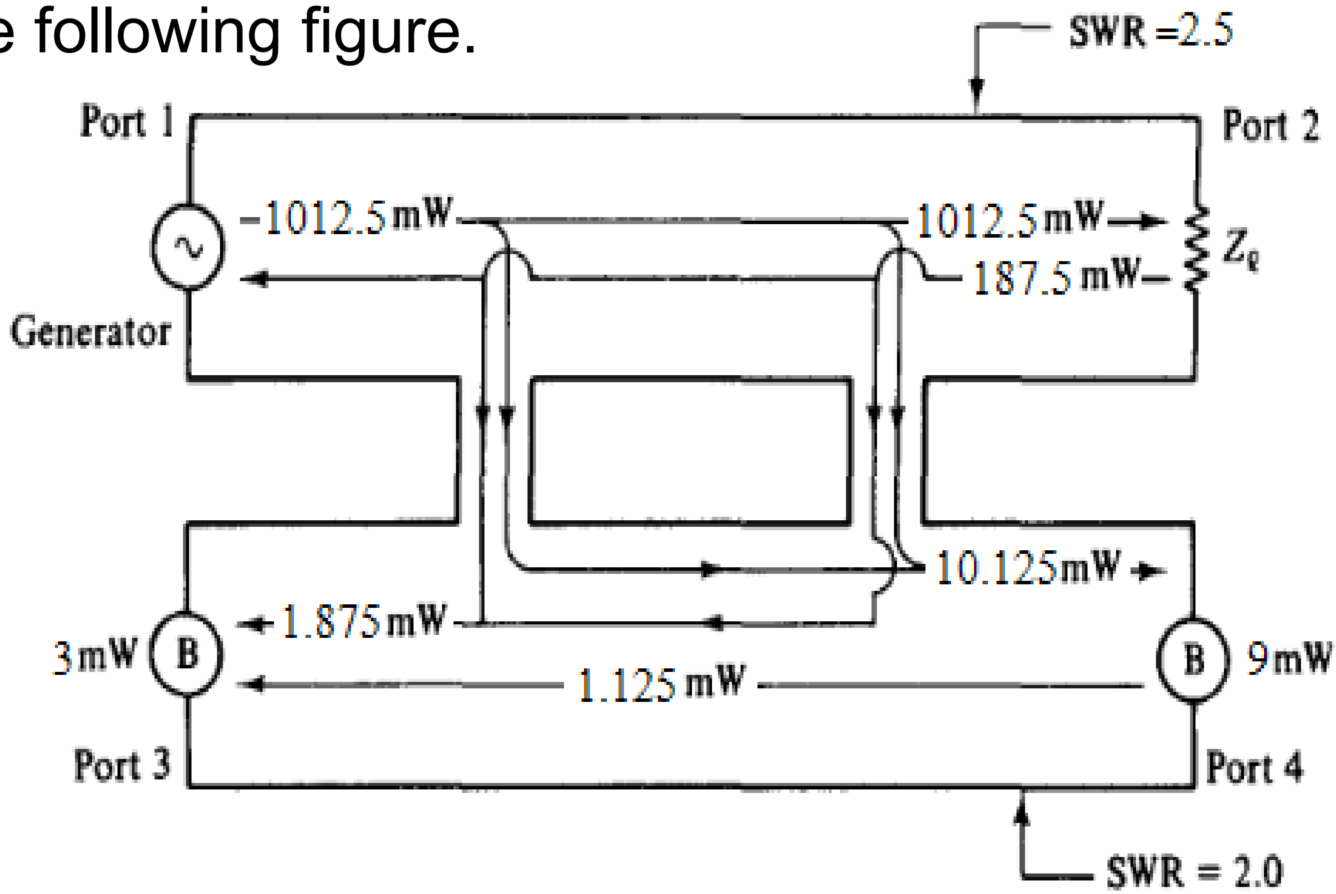
Directional Coupler

A symmetrical directional coupler has an infinite directivity and a forward attenuation of 20 dB. The coupler is used to monitor the power delivered to a load Z_L as shown in the following figure. Bolometer 1 introduces a VSWR of 2 on arm 4; bolometer 2 is matched to arm 3. If bolometer 1 reads 9 mW and bolometer 2 reads 3 mW, find (i) the amount of power dissipated in the load (ii) VSWR of arm 2.



Directional Coupler

The wave propagation in the directional coupler is shown in the following figure.



Isolator

- A two-port non-reciprocal device
- Produces a minimum attenuation to wave in one direction and very high attenuation in the opposite direction.
- Isolators are generally used to improve the frequency stability of microwave generators
- Isolators can be made by inserting a ferrite (non-metallic materials with very high resistivity ρ, nearly 10^{14} times greater than metals and dielectric constants ε_r is in between 10-15 and relative permeability of the order of 1000) rod along the axis of a rectangular waveguide.

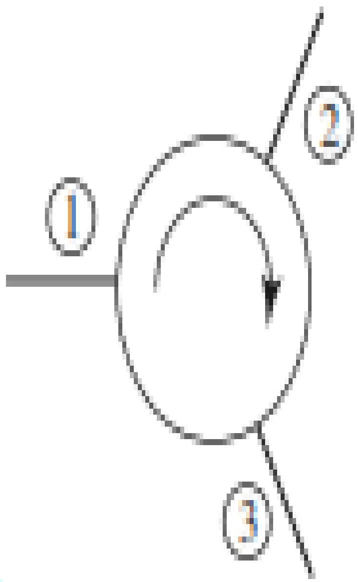
Circulator

- A circulator is a multiport junction in which the wave can travel from one port to next immediate port in one direction only.
- Three-port and four-port microwave circulators are the most common.

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

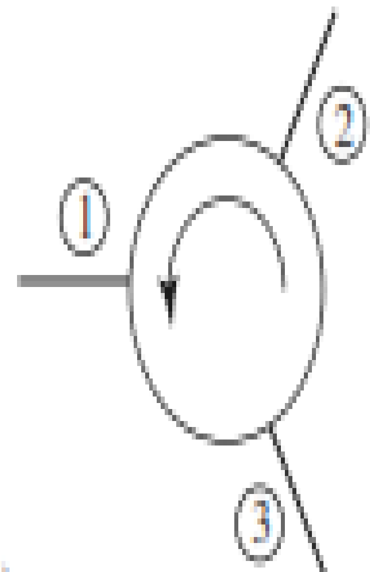
Circulator

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



(a)

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



(b)

Week 11

Slide 206-218

EEE 4181

MICROWAVE ENGINEERING

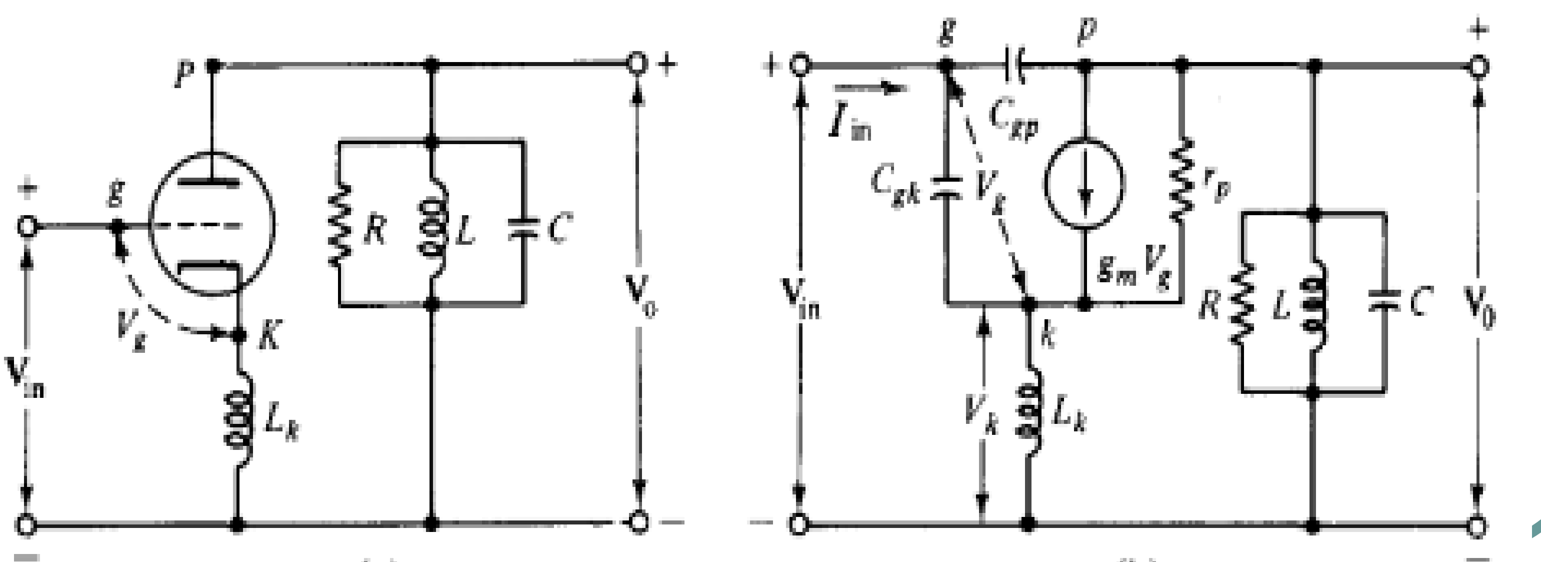
PART 4:

Microwave Tubes

Limitations of Conventional Tubes

Conventional tubes cannot be used as active devices at frequencies above 100MHz because of ……………

1. Lead Inductance and Interelectrode capacitance effect



(a) (b)

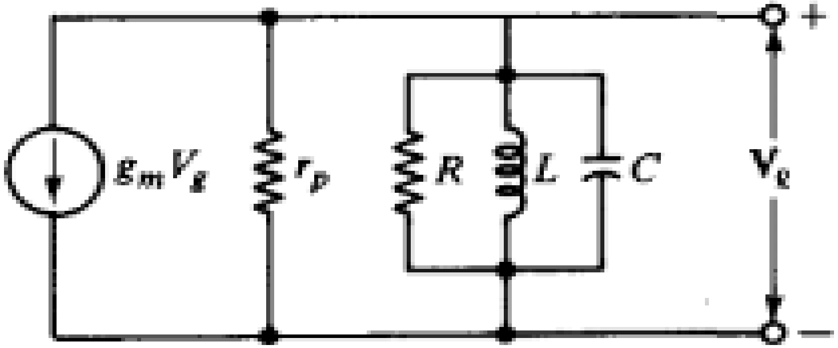
Limitations of Conventional Tubes

2. Transit time effect

Below microwave range, transit time is insignificant compared to the signal's time period. However, at high frequencies, transit time becomes an appreciable portion of the signal cycle. Consequently, on its travel to plate from the cathode, the electron will oscillate back and forth in the cathode-grid space. The overall result of transit time effects is to reduce the operating efficiency of the conventional tube.

Limitations of Conventional Tubes

Higher gain can only be achieved at the cost of bandwidth.



$$V_l = \frac{g_m V_g}{G + j\left[\omega C - 1/(\omega L)\right]}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_l = \frac{G}{2C} - \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}}$$

$$BW = \omega_h - \omega_l = \frac{G}{C}$$

$$\omega_h = \frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}}$$

$$Gain \text{ x } BW = \frac{V_l}{V_g} \text{ x } \frac{G}{C} = \frac{g_m}{C}$$

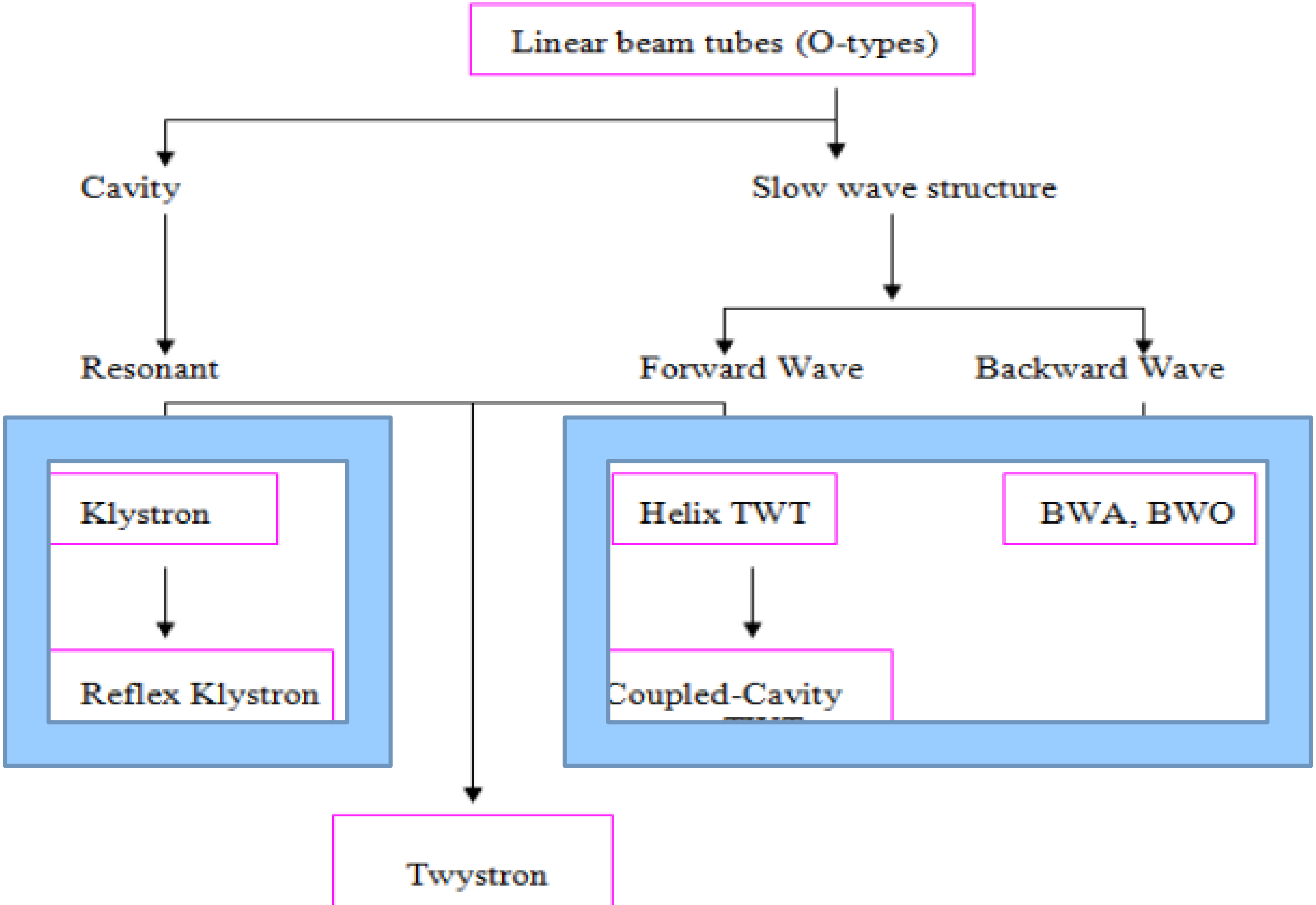
Microwave Tubes

Efficient Microwave tubes usually operate on the theory of electron **velocity modulation** concept

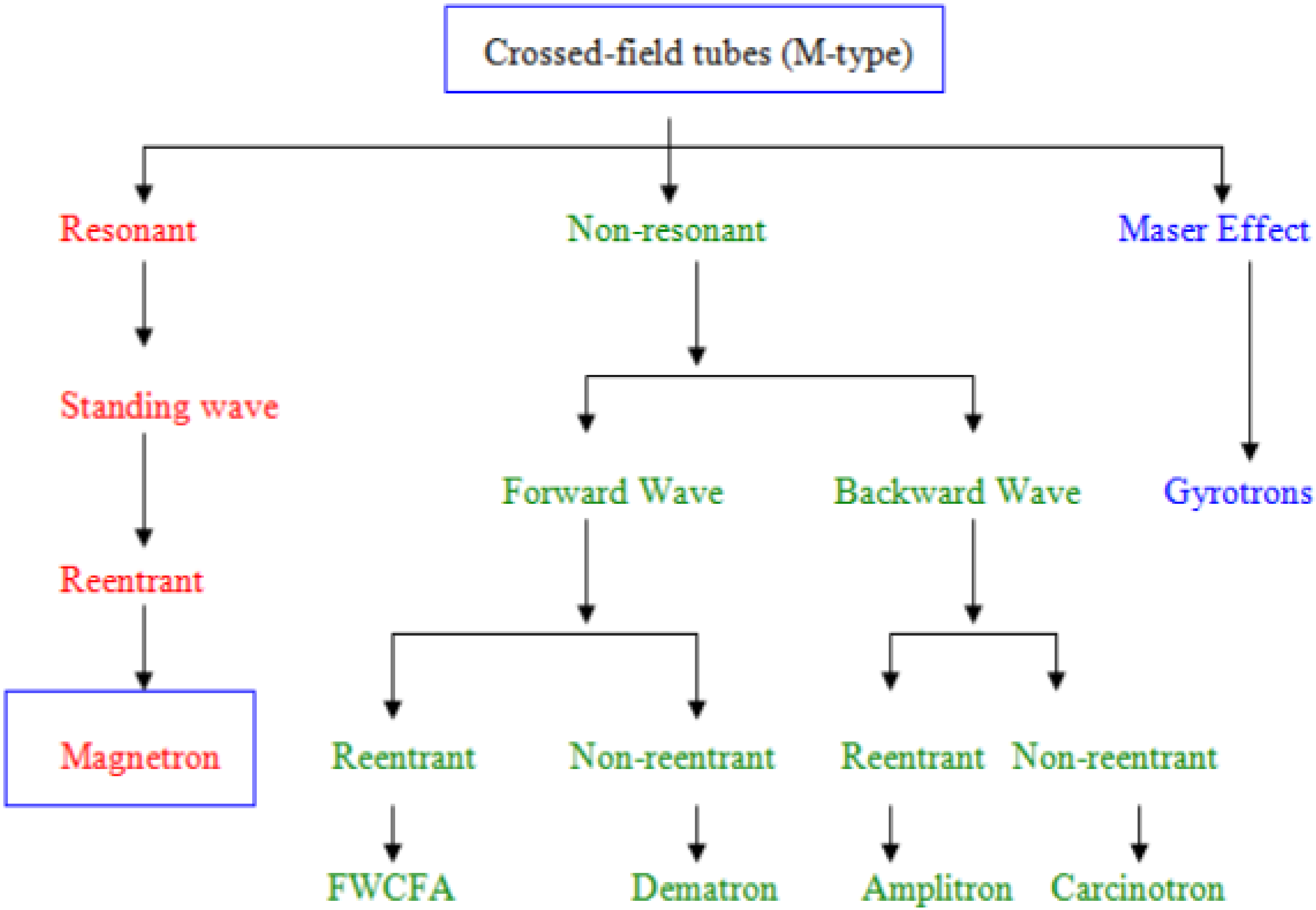
These microwave tubes can be grouped into two categories:

1) In linear beam, or O-type tubes electron beam traverses the length of the tube and is parallel to the electric field.
2) In the crossed-field, or M-type tube the beam focusing magnetic field is perpendicular to the accelerating electric field.

O-type Microwave Tubes



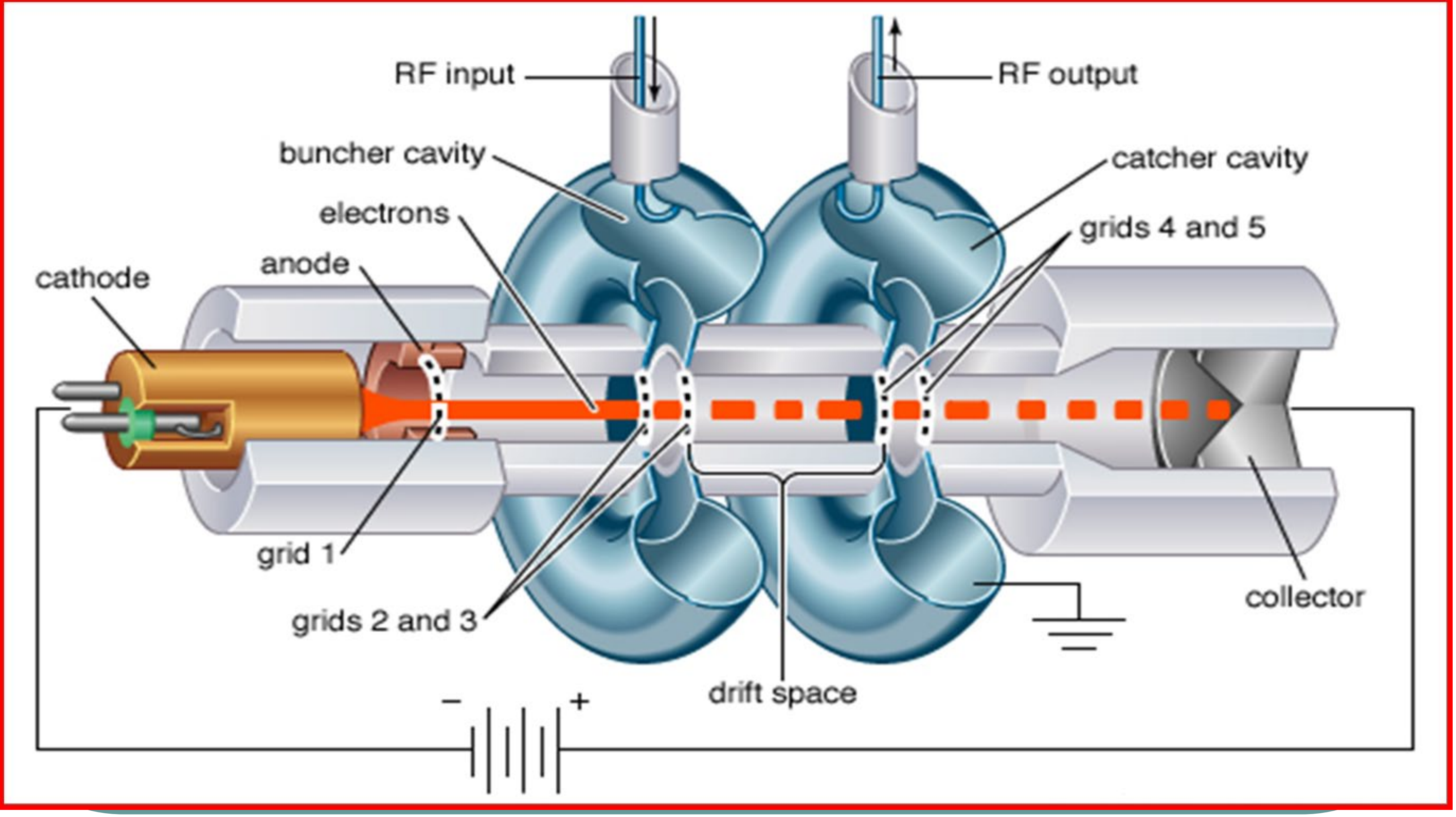
M-type Microwave Tubes



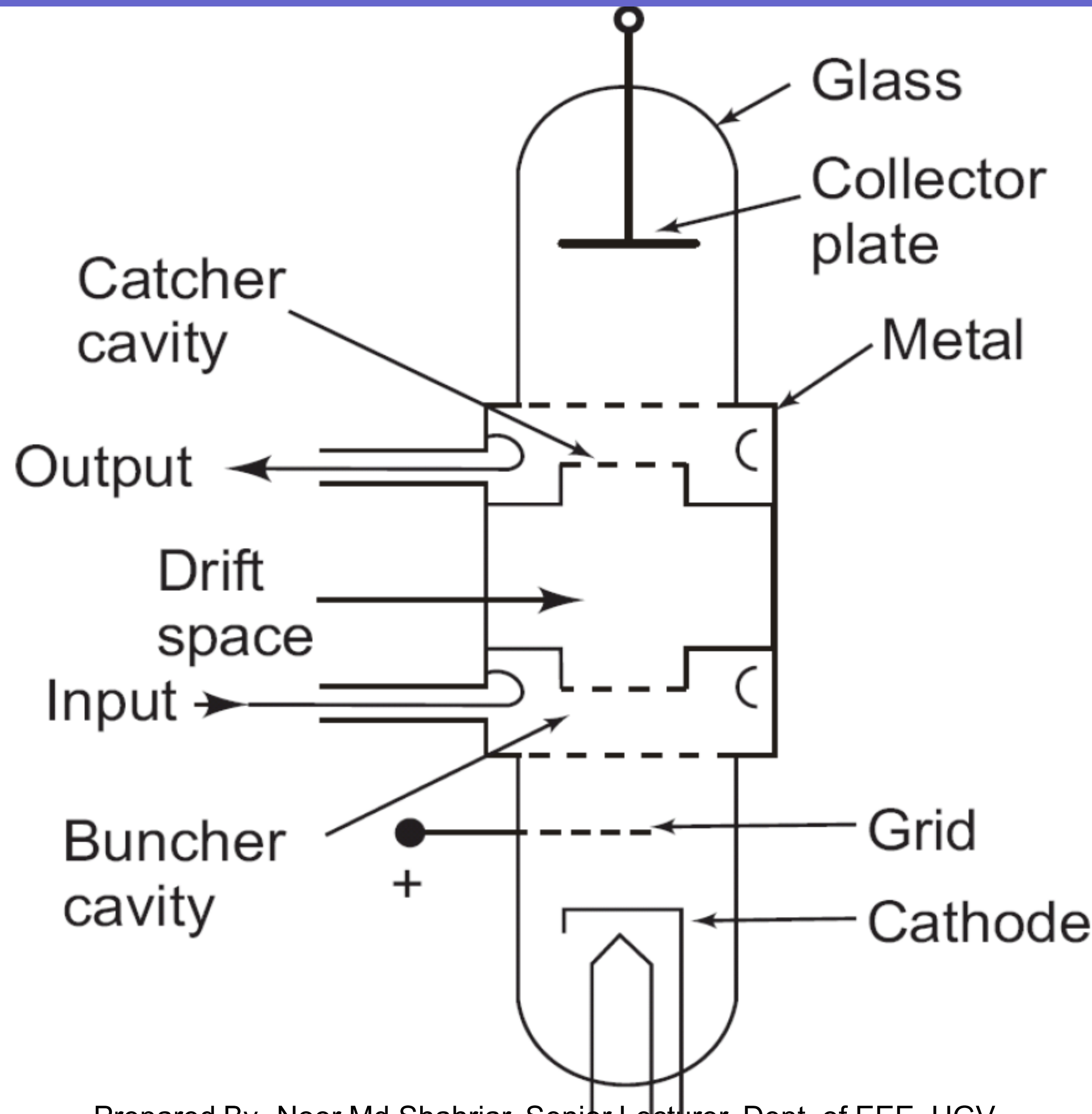
Klystron

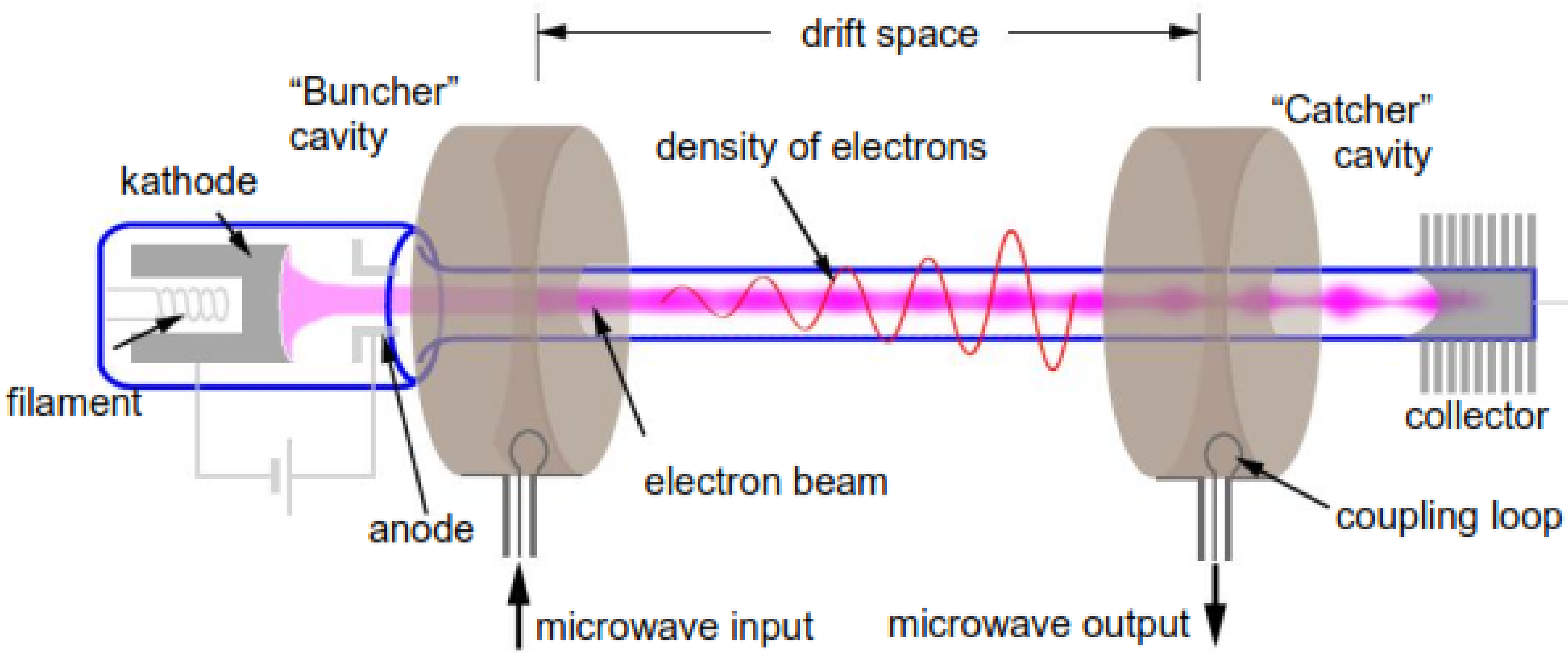
- In microwave region, performs the functions of generates, receives and amplifies signals
- Configurations:
  - Reflex – low power microwave oscillator
  - Multicavity – low power microwave amplifier

Two Cavity Klystron



Two Cavity Klystron



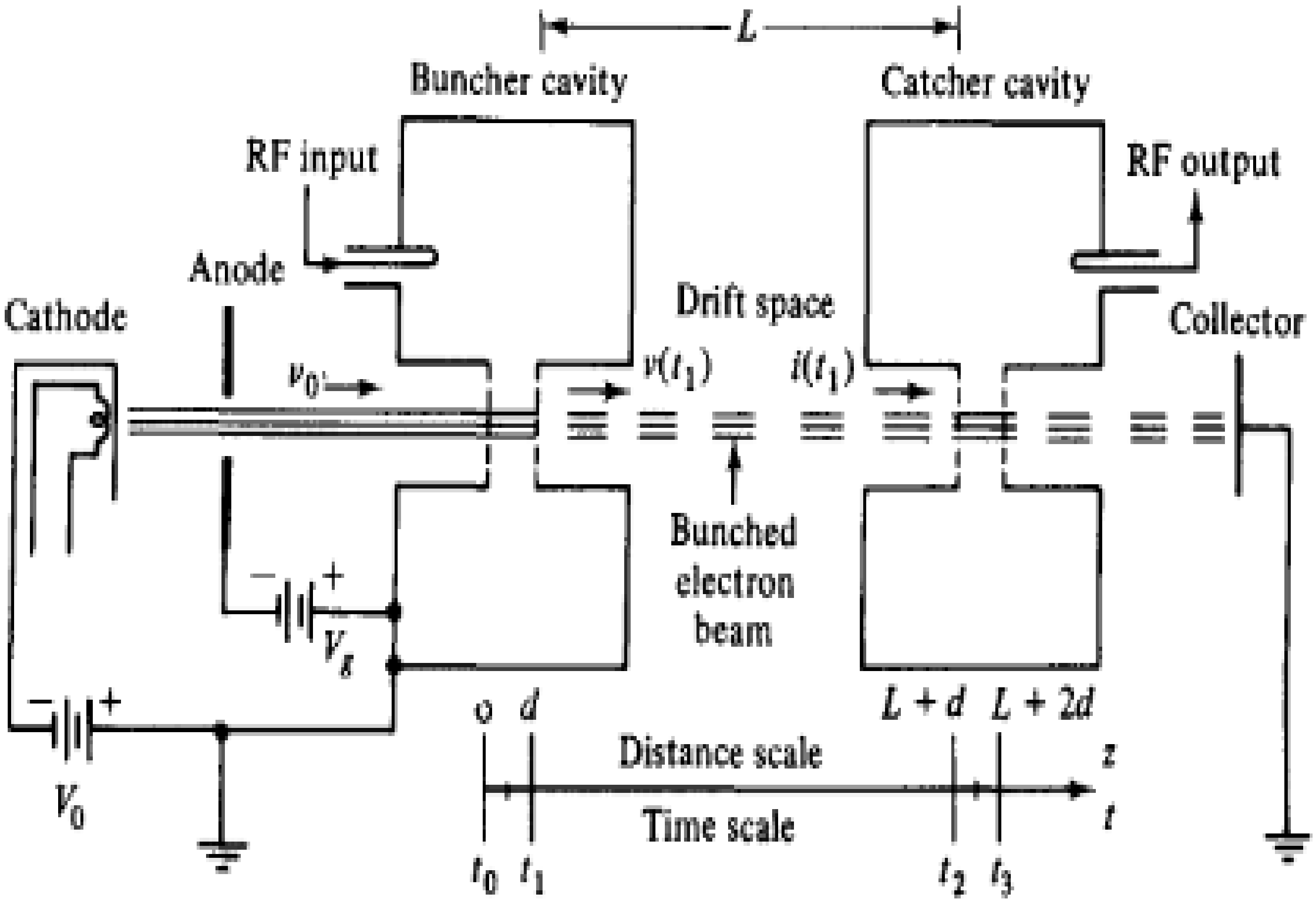

drift space
"Buncher"
cavity
kathode
density of electrons
"Catcher"
cavity
filament
collector
electron beam
anode
coupling loop
microwave input
microwave output

Two Cavity Klystron

Working Principle:

1) The electrons emitted from the cathode forms an electron beam that accelerated towards the buncher or input cavity by the high dc voltage V_0
2) RF signal in the input cavity alternately accelerate and decelerate the electrons of the beam passing through the grids of the cavity
3) The electrons form the bunches in the drift space
4) On entering catcher or output cavity, these electron bunches induce RF signal of the same resonant frequency. Since the largest concentration of electrons is in the bunch, an enormous transfer of energy is possible in the output cavity.

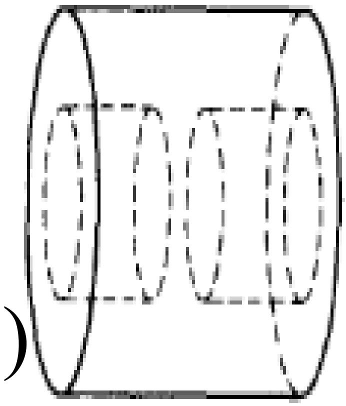
Functional diagram of Klystron

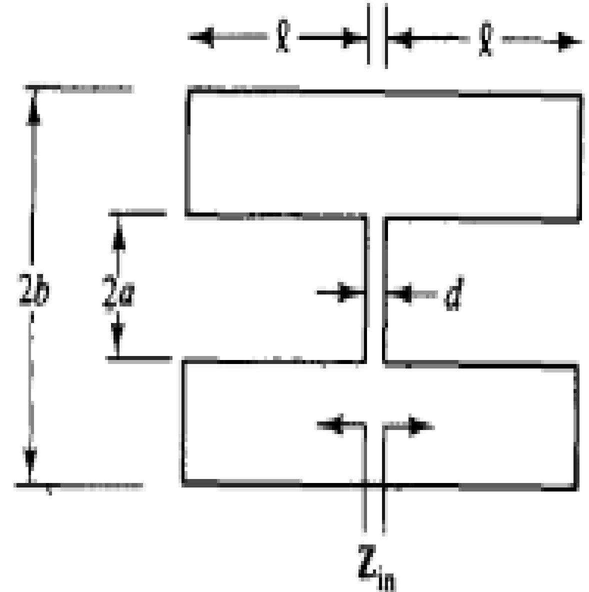


Week 12

Slide 220-234

Reentrant Cavity





$$Z_{in} = jZ_0 \tan(\beta l)$$

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{b}{a}$$

$$L = \frac{1}{\pi\omega}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{b}{a}\tan(\beta l)$$

$C_g=(\varepsilon\pi a^2/d)$

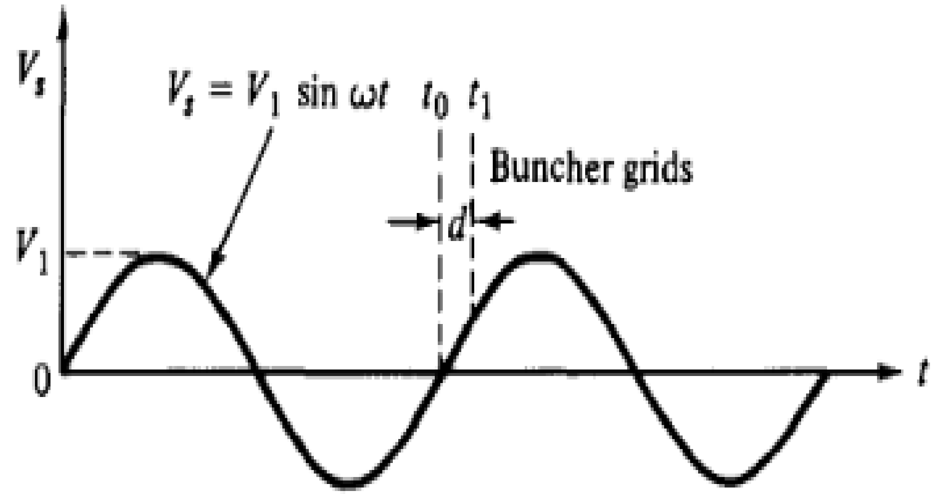
At resonance $\omega L=1/(\omega C_g)$.

$$\boxed{\tan(\beta l) = \frac{dv}{\omega a^2 \ln(b/a)}}$$

$$v = \sqrt{\frac{1}{\mu\varepsilon}}$$

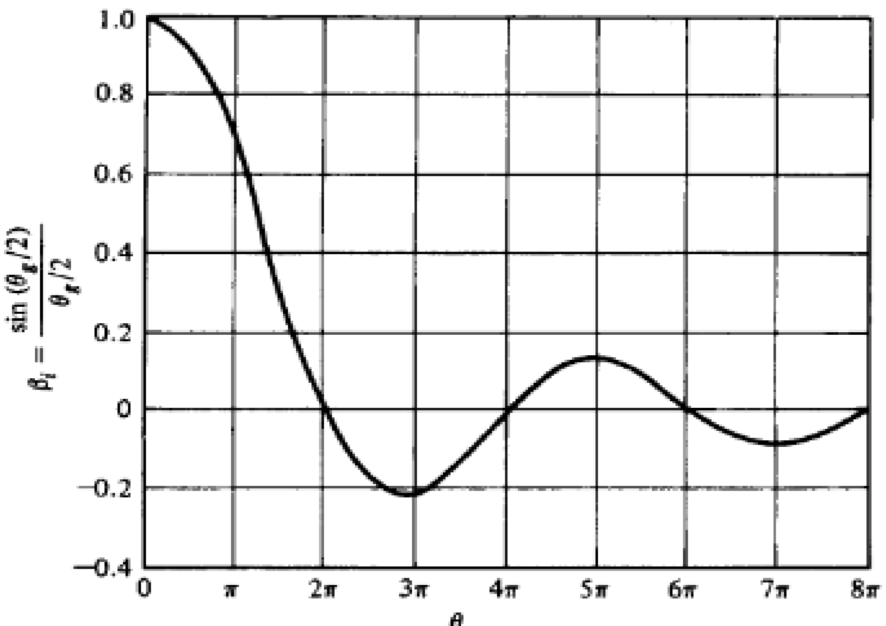
Velocity modulation in Klystron

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0}.$$



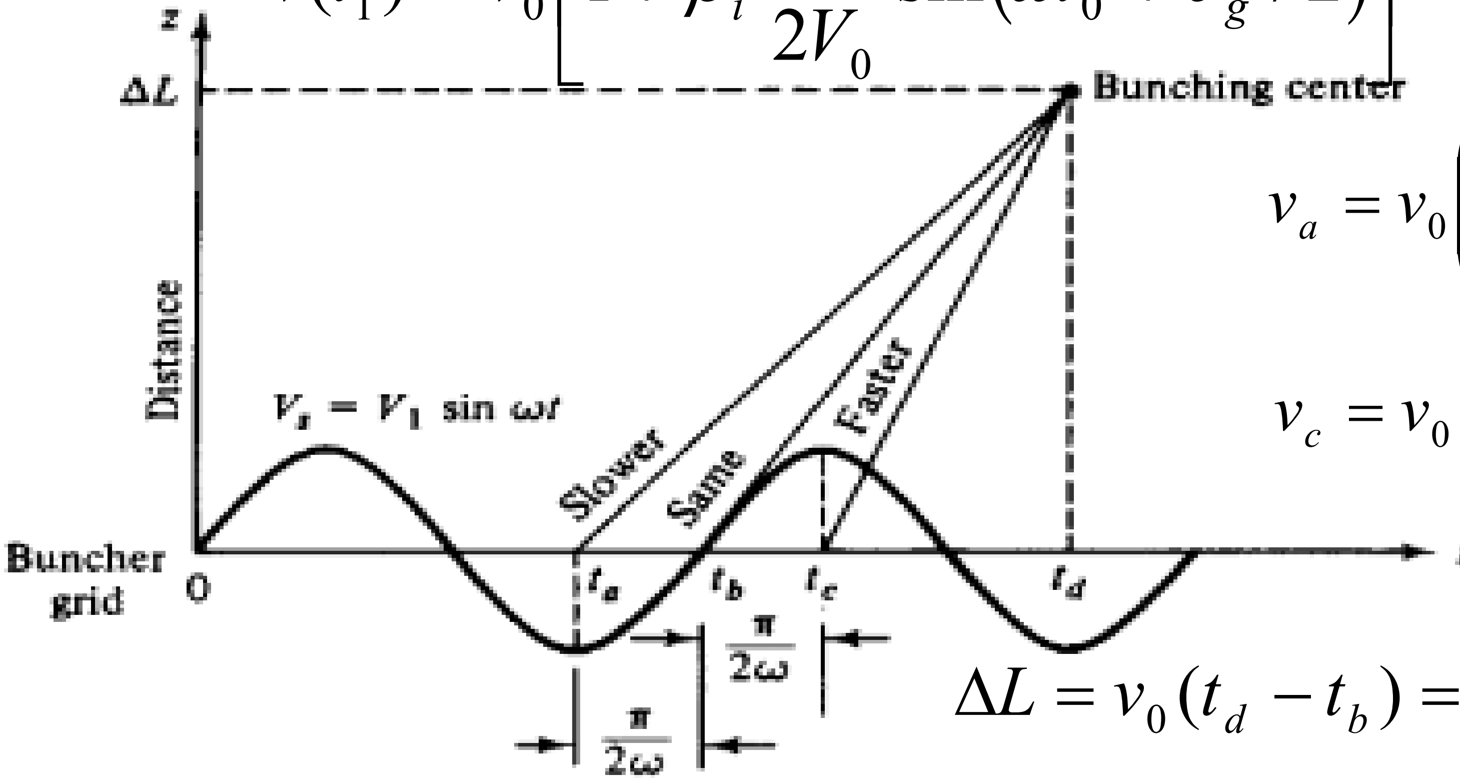
$$\langle V_s \rangle = V_1 \beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2} \right)$$

$$\beta_i \equiv \frac{\sin\left(\frac{\theta_g}{2}\right)}{\left(\frac{\theta_g}{2}\right)}$$



Bunching mechanism in Klystron

$$v(t_1) = v_0\left[1 + \beta_i \frac{V_1}{2V_0}\sin\left(\omega t_0 + \theta_g / 2\right)\right]$$



$$v_a = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right).$$

$$v_c = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right).$$

$$\Delta L = v_0(t_d - t_b) = v_0 \frac{\pi V_0}{\beta_i \omega V_1}.$$

Bunching mechanism in Klystron

$$\Delta L = v_0(t_d - t_b)$$

$$\Delta L = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right)(t_d - t_c) = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right)(t_d - t_b - \frac{\pi}{2\omega})$$

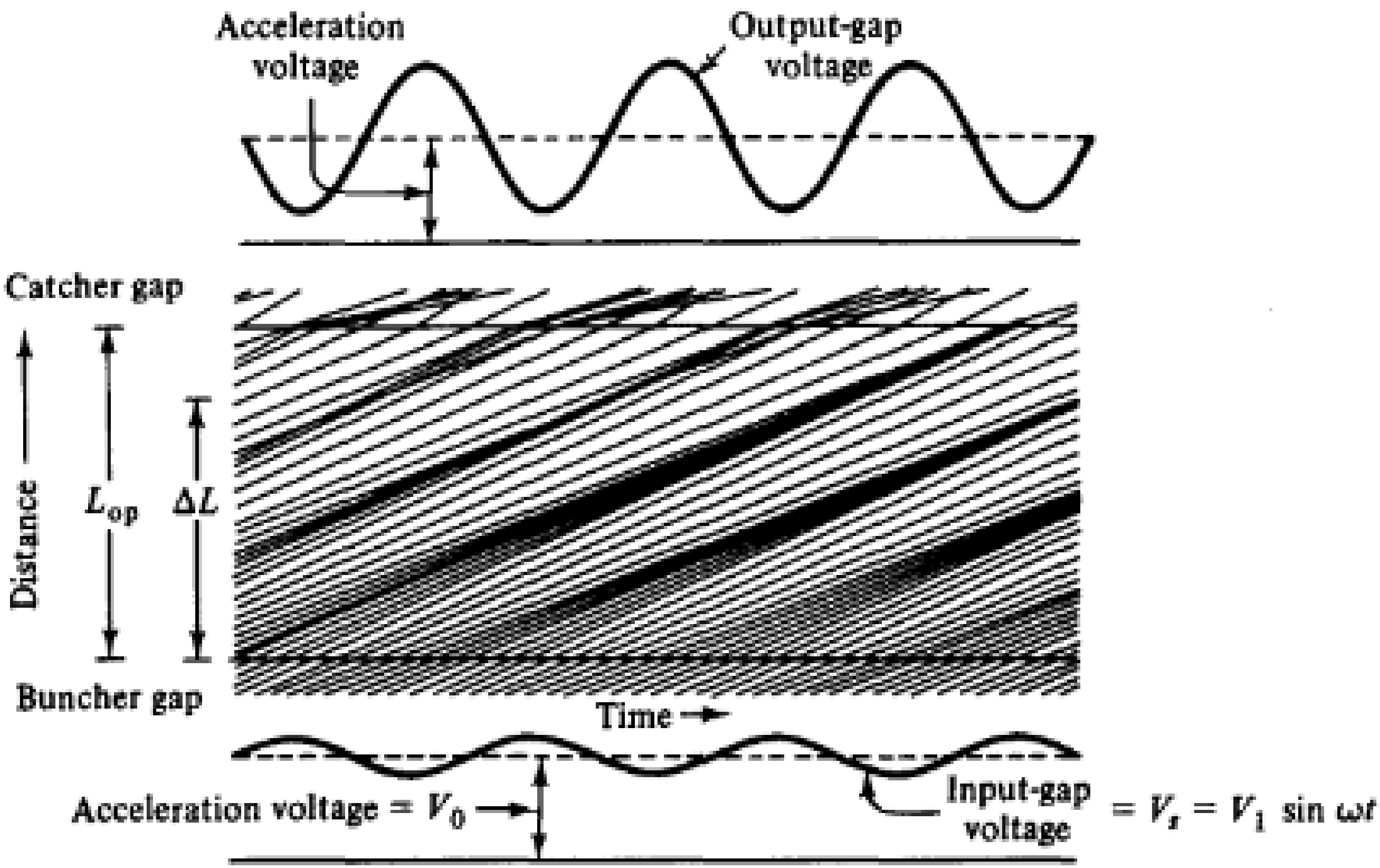
$$= v_0(t_d - t_b) - v_0\left[\frac{\pi}{2\omega} - \frac{\beta_i V_1}{2V_0}(t_d - t_b) + \frac{\beta_i V_1 \pi}{4V_0\omega}\right]$$

$$\Delta L = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right)(t_d - t_a) = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right)(t_d - t_b + \frac{\pi}{2\omega})$$

$$= v_0(t_d - t_b) + v_0\left[\frac{\pi}{2\omega} - \frac{\beta_i V_1}{2V_0}(t_d - t_b) - \frac{\beta_i V_1 \pi}{4V_0\omega}\right]$$

$$\Delta L = v_0(t_d - t_b) = v_0 \frac{\pi V_0}{\beta_i \omega V_1}.$$

Bunching mechanism in Klystron



Two-Cavity Klystron

- What should the spacing be between the two cavities in order to achieve a maximum degree of bunching?
- What magnitude of current is induced in the output cavity? $I_0 dt_0 = i_2 dt_2$

$$T = t_2 - t_1 = \frac{L}{v(t_1)} = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$\Rightarrow t_2 = t_1 + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

$$\Rightarrow dt_2 / dt_0 = 1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

Two-Cavity Klystron

$$i_2 = I_0 \frac{dt_0}{dt_2} = I_0\left[1 + X\cos\left(\omega t_0 + \theta_g / 2\right)\right]$$

Beam current in catcher cavity:

$$\therefore i_2 = I_0 + 2I_0 \sum_{n=1}^{\infty} J_n(nX)\cos\left[n\omega(t_2 - \tau - T_0)\right].$$

Magnitude of the fundamental component of beam current:

$$I_{2f} = 2I_0 J_1(X)$$

Magnitude of induced current in catcher cavity:

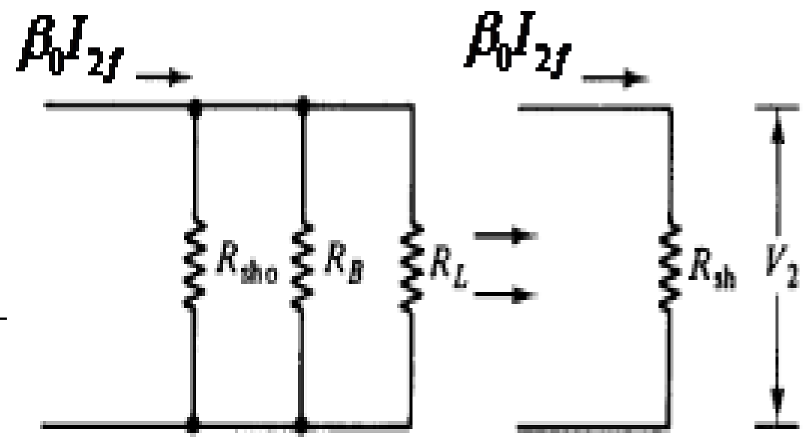
$$\boxed{I_{2ind} = \beta_0 I_{2f} = \beta_0 2I_0 J_1(X).}$$

Two-Cavity Klystron

Induced current in catcher cavity is maximum when $J_1(X)$ is maximum; this occurs when X=1.841. At this X, $J_1(X)$ =0.582. Now from $X=(\beta_i V_1 \theta_0)/(2V_0)$, we obtain

$$L_{optimum} = \frac{v_0}{\omega}\theta_0 = \frac{2XV_0 v_0}{\beta_i \omega V_1} = \frac{3.682V_0 v_0}{\beta_i \omega V_1}$$

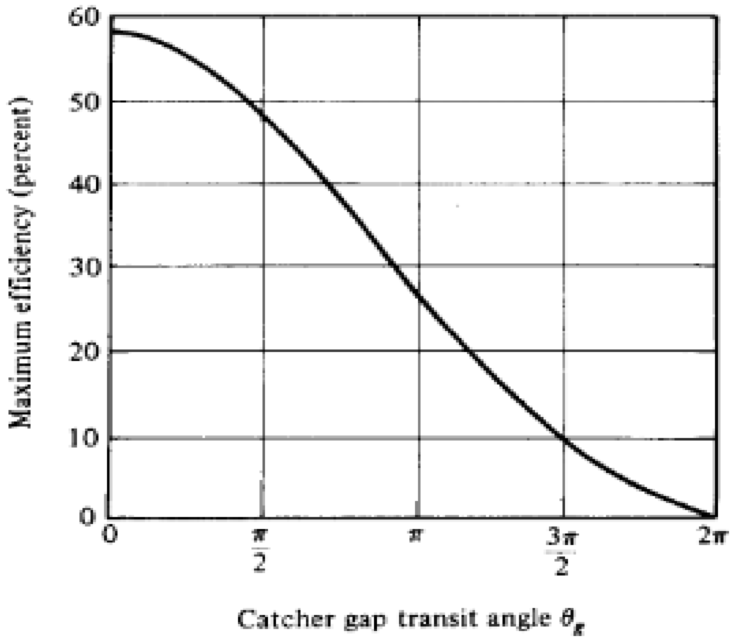
Two-Cavity Klystron



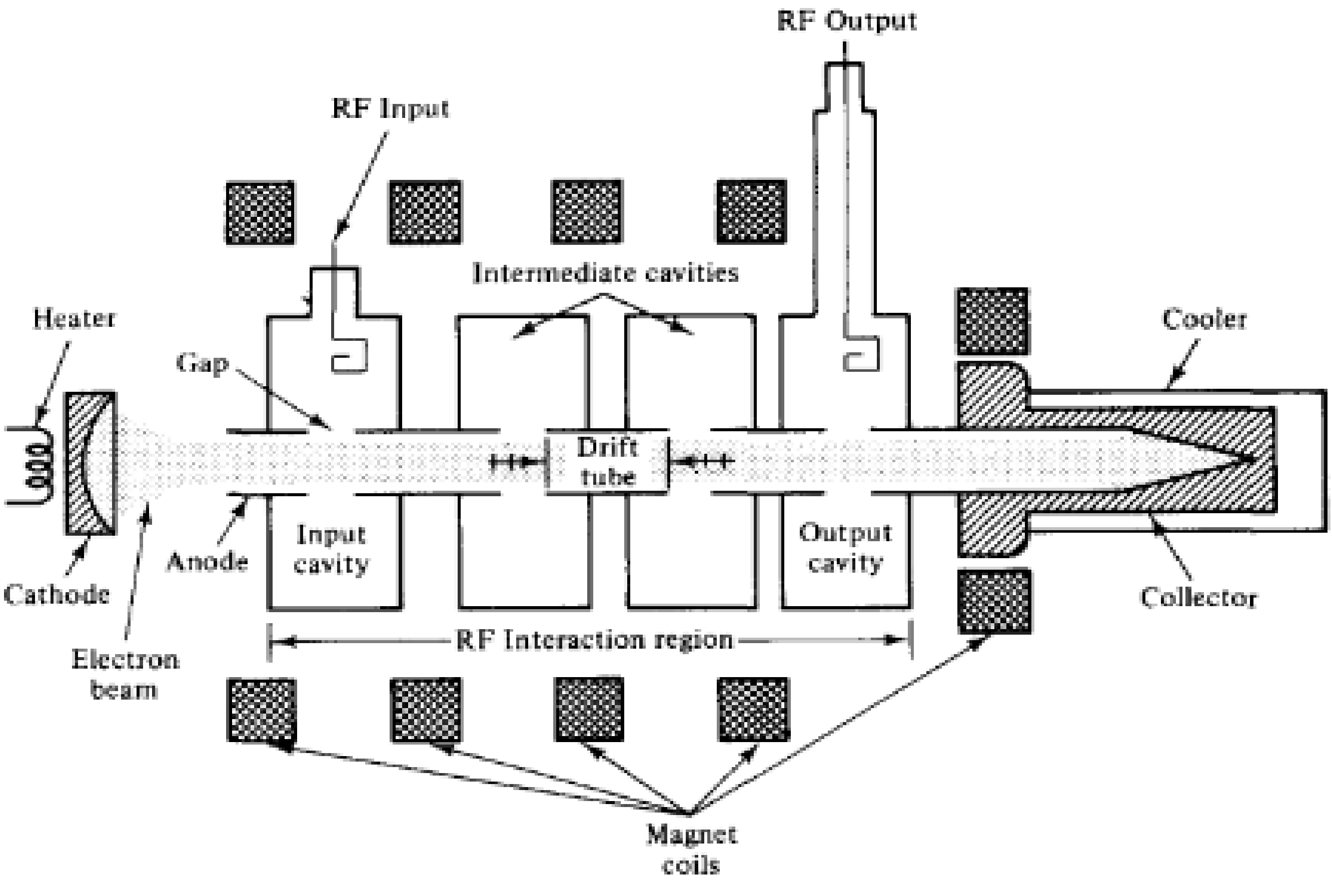
$$A_v \equiv \frac{|V_2|}{|V_1|} = \frac{\beta_0 I_{2f} R_{sh}}{V_1} = \frac{\beta_0^2 \theta_0 J_1(X) R_{sh}}{R_0 X}$$

$$P_{out} = \frac{(\beta_0 I_{2f})^2}{2} R_{sh} = \frac{\beta_0 I_{2f} V_2}{2}.$$

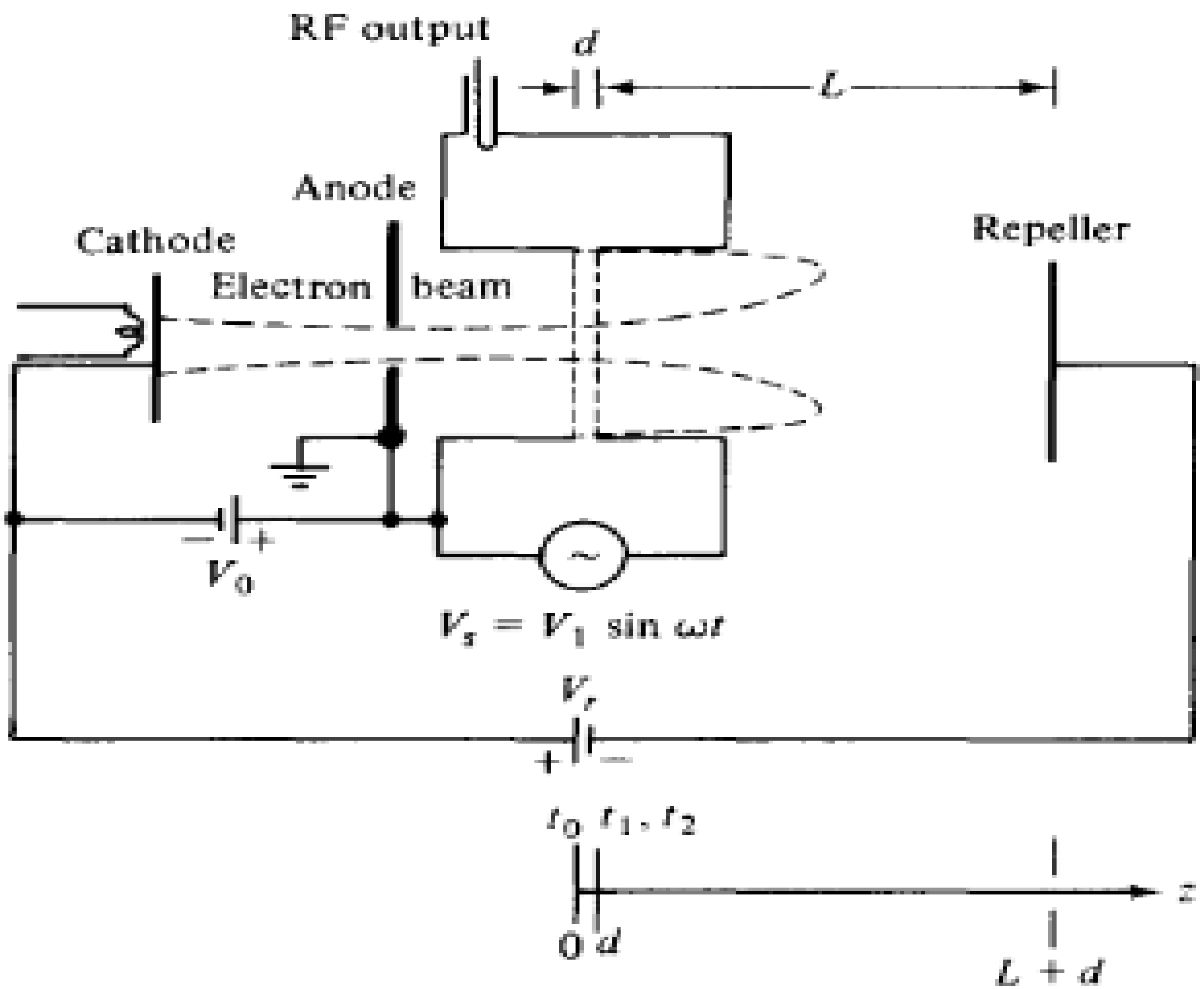
$$Efficiency = \frac{\beta_0 I_{2f} V_2}{2V_0 I_0} = \frac{\beta_0 J_1(X) V_2}{V_0}$$



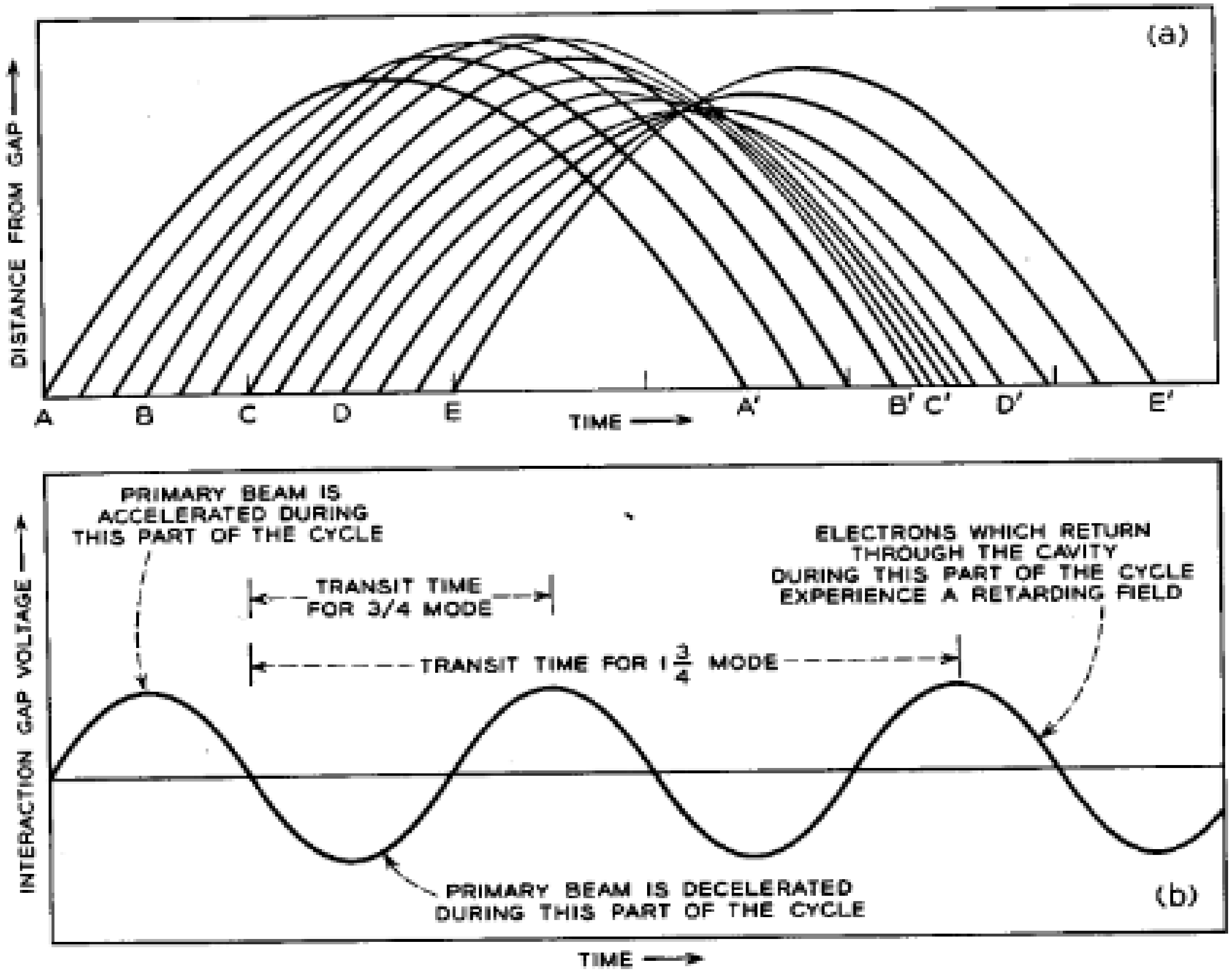
Multi-Cavity Klystron



Reflex Klystron



Reflex Klystron



Reflex Klystron

$$m\frac{d^2z}{dt^2} = -e\frac{V_0+V_r}{L}$$

$$\therefore \frac{dz}{dt} = -e\frac{V_0+V_r}{mL}\int_{t_1}^{t} dt = -e\frac{V_0+V_r}{mL}(t-t_1)+K_1$$

$$z = -e\frac{V_0+V_r}{mL}\int_{t_1}^{t}(t-t_1)dt + v(t_1)\int_{t_1}^{t} dt = -e\frac{V_0+V_r}{2mL}(t-t_1)^2 + v(t_1)(t-t_1)+K_2$$

$$z = -e\frac{V_0+V_r}{2mL}(t-t_1)^2 + v(t_1)(t-t_1)+d$$

Since at $t=t_2$, $z=d$

$$0 = -e\frac{V_0+V_r}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1)$$

$$\boxed{v(t_1) = v_0\left[1+\beta_i\frac{V_1}{2V_0}\sin\left(\omega t_0+\theta_g/2\right)\right]}$$

$$T' = (t_2-t_1) = \frac{2mL}{e(V_0+V_r)}v(t_1) = T_0'\left[1+\beta_i\frac{V_1}{2V_0}\sin\left(\omega t_0+\theta_g/2\right)\right]$$

Reflex Klystron

$$i_2 = -I_0 - 2I_0 \sum_{n=1}^{\infty} J_n(nX') \cos\left[n\omega(t_2 - \tau - T_0')\right]$$

$$\text{Efficiency} \equiv \frac{\mathrm{P_{ac}}}{\mathrm{P_{dc}}} = \frac{X' J_1(X')}{\pi N}$$

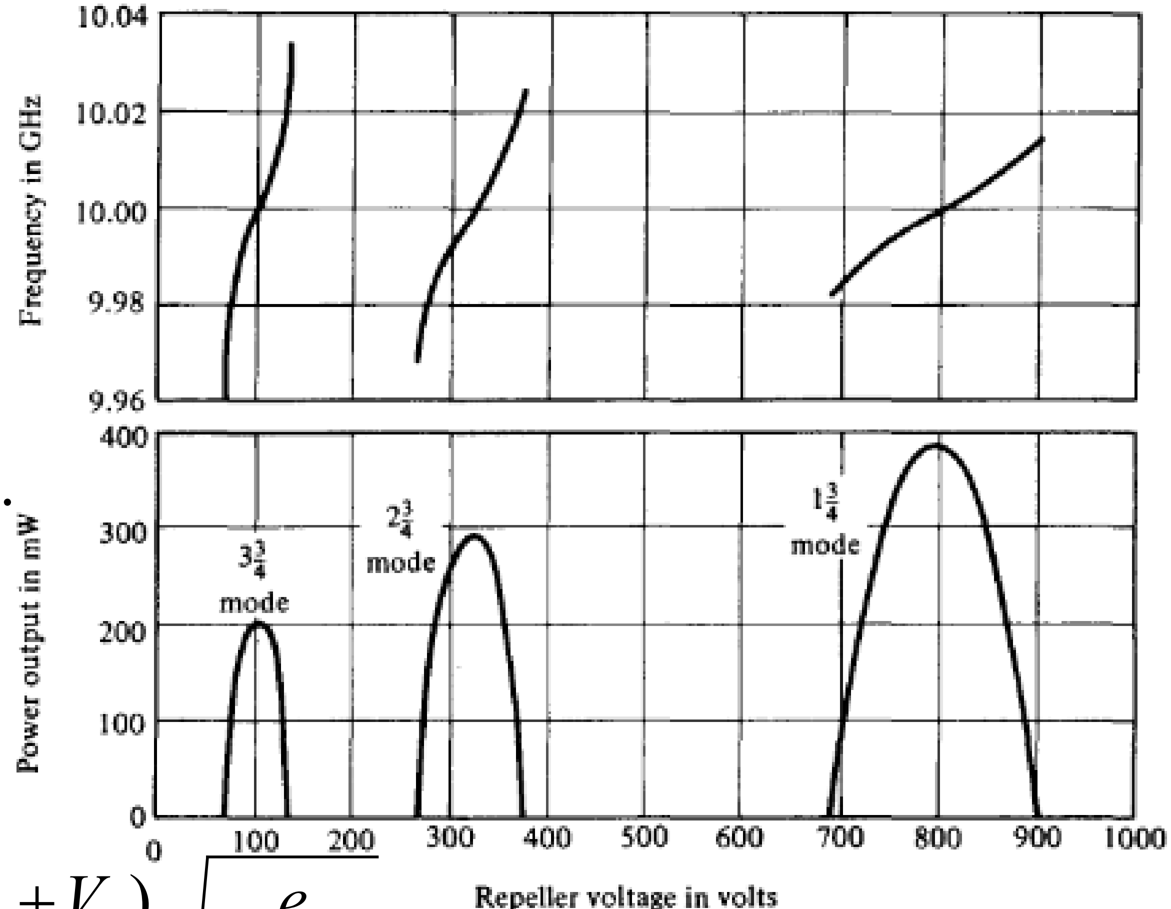
For N=1¾ mode (n=2), the power of the reflex klystron is maximum; and it occurs at X'=2.408 when $J_1(X')=0.52$. The maximum efficiency of a reflex klystron is then

$$\text{Efficiency} = \frac{X' J_1(X')}{\pi N} = \frac{2.408 \times 0.52}{\pi(1.75)} = 22.77\%.$$

Effect of repeller voltage on operating frequency and output power

$$f = \frac{N(V_0 + V_r)}{6.74L\sqrt{V_0}} 10^6 \, Hz.$$

$$df = \frac{N10^6}{6.74L\sqrt{V_0}} dV_r \, Hz.$$



$$P_{ac} = \frac{V_0 I_0 X' J_1(X')(V_0 + V_r)}{\omega L} \sqrt{\frac{e}{2mV_0}}.$$

Week 13

Slide 236-250

Traveling Wave Tube

Broadband High average power amplifier.

Used in-

- Medium – power satellite
- Higher – power satellite transponder output.

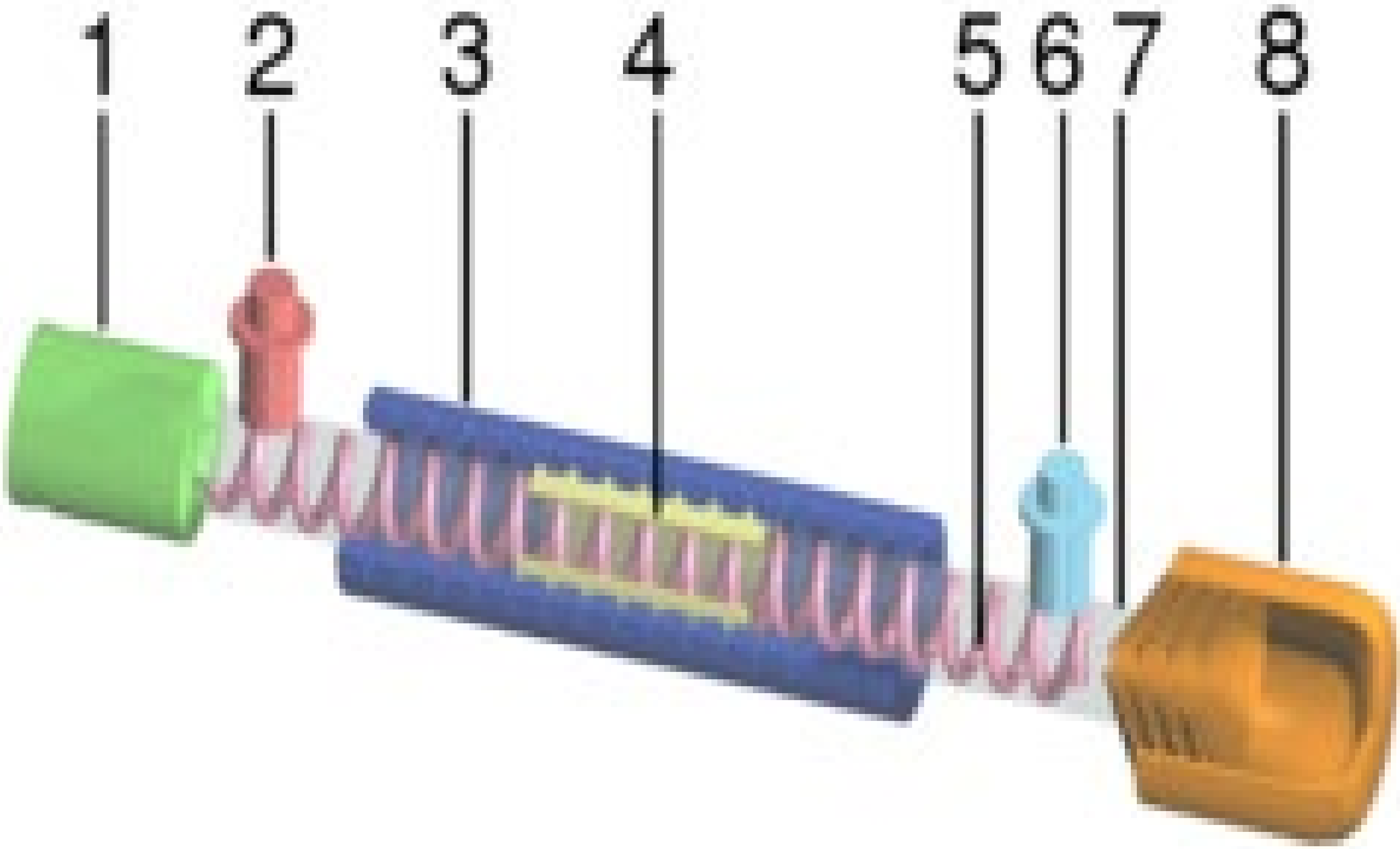
Specifications

- Frequency Range: 3 GHz and higher
- Bandwidth: about 0.8 GHz
- Efficiency: 20 to 40%
- Power Output: up to 10kW average
- Power gain: up to 60dB

Comparison of TWTA and Klystron Amplifier

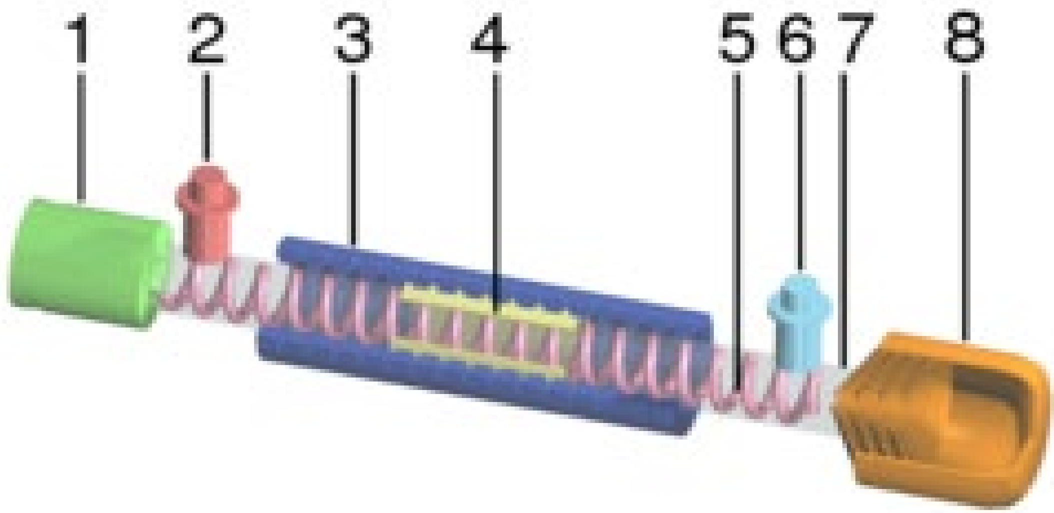
Klystron Amplifier	TWT
1. Linear beam or ‘O’ type Device	1. Linear beam or ‘O’ type device
2. Uses Resonant cavities for input and output circuits	2. Uses non resonant wave circuits
3. Narrowband device	3. Wideband device
4. Beam interacts with standing wave in the cavities only	4. Beam interacts with the traveling wave through out the tube

Cutaway view of a HELIX TWT



TWT

1. Electron Gun
2. RF input
3. Magnets
4. Attenuator
5. Helix Coil
6. RF Output
7. Vacuum tube
8. Collector



TWT

- Electron Gun produces and then accelerates an electron beam along the axis of the tube.
- The surrounding magnet provides a static magnetic field along the axis of the tube to focus the electrons into a tight beam.
- A longitudinal helix slow wave non-resonant guide is placed at the centre of the tube that provides a low impedance transmission line for the RF energy within the tube.

TWT

- The TWT is designed with a **helix delay structure** to slow the traveling wave down to or below the speed to the electrons in the beam.
- The RF signal injected at the input end of the helix travels down the helix wire at the speed of light but the **coiled shape** helix causes the wave velocity to reduce at the beam velocity in the axial direction.
- Helix provides **maximum interaction** between the fields and the moving electrons to form bunching.

TWT

- As the bunches release energy to the signal on the helix, amplification begins.
- This amplified signal causes a denser electron bunch which in turn amplifies the signal even more.
- This process continues as the RF wave and the electron beam travel down the length of the tube.

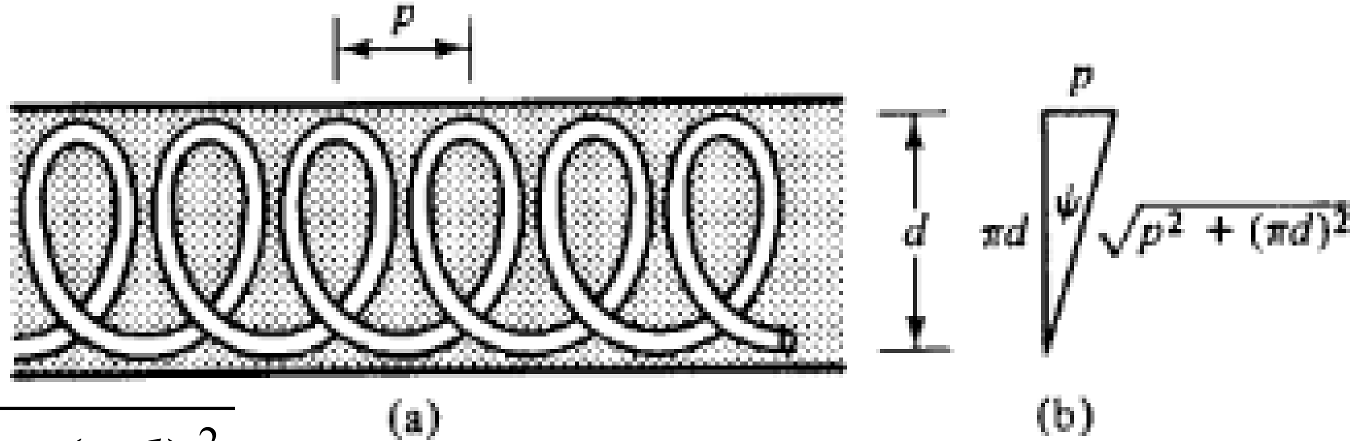
Why attenuator?

- An attenuator is placed over a part of the helix on midway to attenuate any reflected waves generated due to the impedance mismatch.
- It is placed after sufficient length of the interaction region so that the attenuation of the amplified signal is insignificant compared to the amplification.

Beam velocity greater than field velocity?

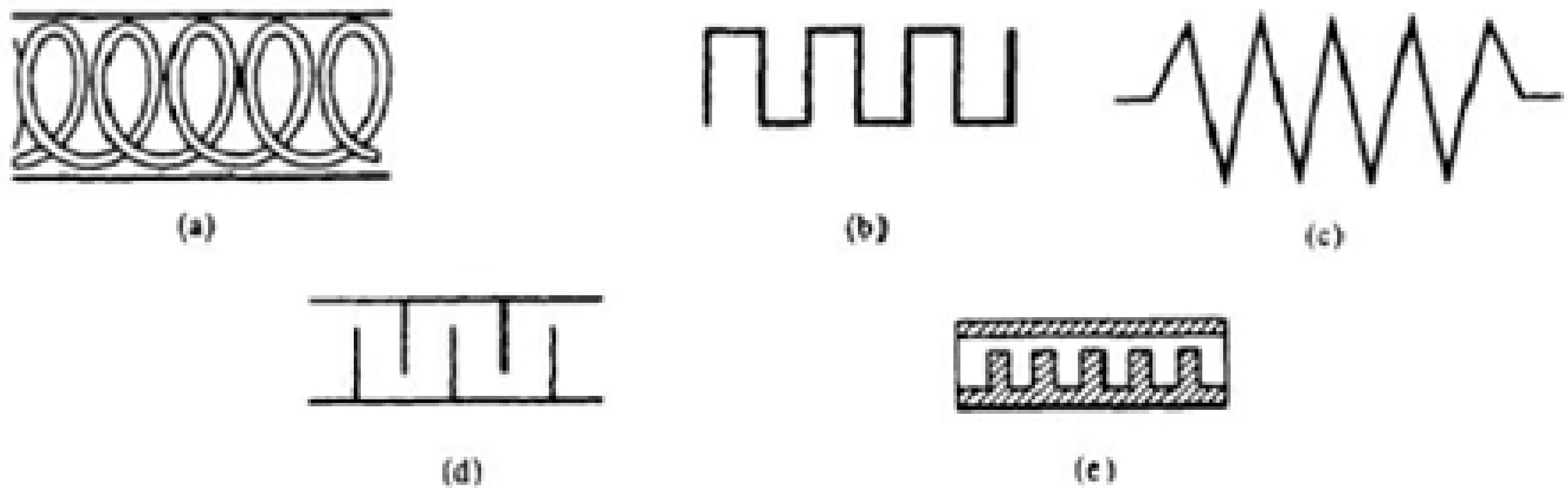
- The dc velocity of the beam is maintained slightly greater than the phase velocity of the traveling wave, so that more electrons face the retarding field than the accelerating field, and a great amount of kinetic energy is transferred from the beam to the electromagnetic field.
- Thus the field amplitude increases forming a more compact bunch and a large amplification of the signal voltage appears at the output of the helix.

Wave Velocity in Helix



$$\frac{\sqrt{p^2 + (\pi d)^2}}{c} = \frac{p}{v_p} \Rightarrow v_p = \frac{pc}{\sqrt{p^2 + (\pi d)^2}} \approx \frac{pc}{\pi d} = \frac{\omega}{\beta}.$$

Changing the number of turns or diameter of the turns in the helix wire, the speed at which RF signal wave travels in the form of **axial E field, can be varied**.



Noted that, to be a slow wave structure, the guiding system must have periodicity.

$$E(x, y, z-L) = E(x, y, z)e^{j\beta_0 L}$$

Floquet's periodicity theorem

TWT- Convection current

$$v = v_0 + v_1 e^{j\omega t - \gamma z}$$

$$\rho = \rho_0 + \rho_1 e^{j\omega t - \gamma z}$$

$$J = -J_0 + J_1 e^{j\omega t - \gamma z}$$

$$E_z = E_1 e^{j\omega t - \gamma z}$$

$$\frac{dv}{dt} = -\frac{e}{m} E_1 e^{j\omega t - \gamma z}$$

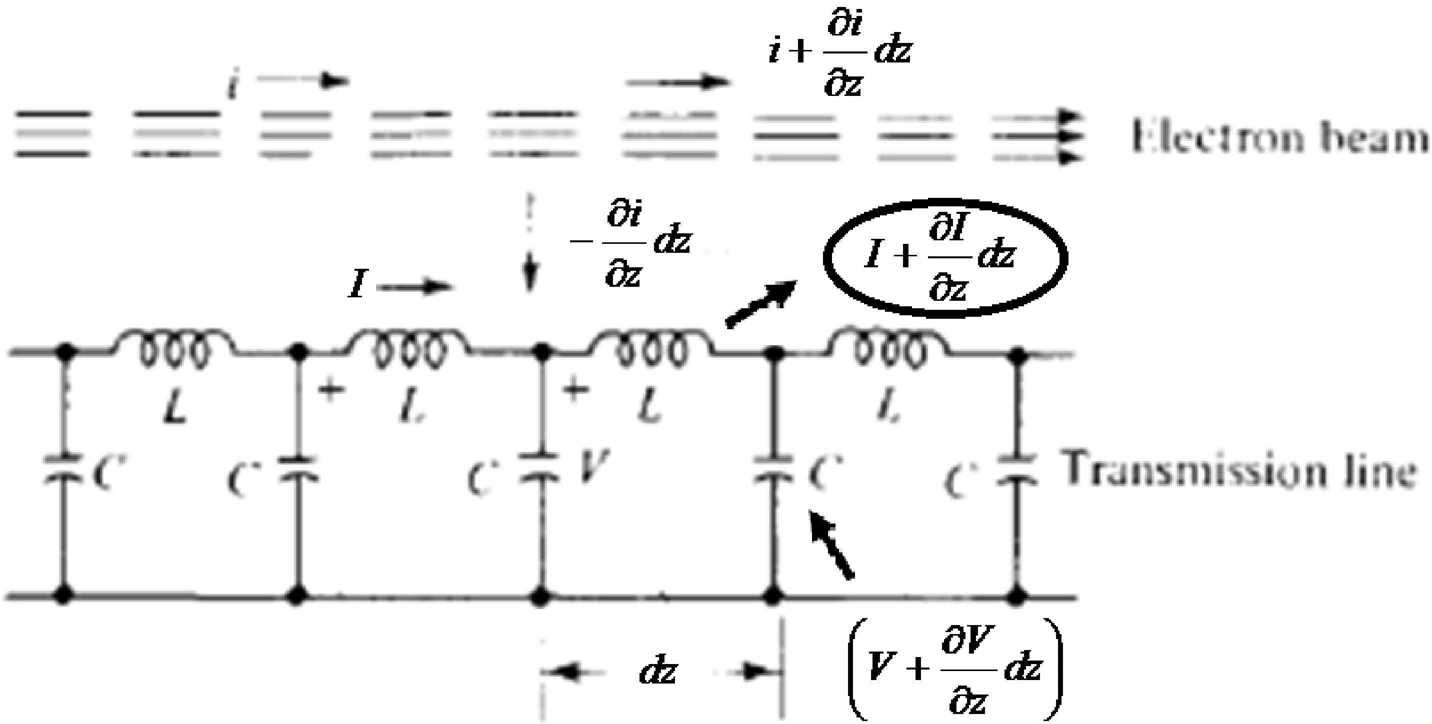
$$v_1 = \frac{-e/m}{j\omega - \gamma v_0} E_1$$

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

$$i = j \frac{\beta_e I_0}{2V_0(j\beta_e - \gamma)^2} E_1$$

Electronic equation

TWT- Axial Electric Field



$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} - \frac{\partial i}{\partial z}$$

$$-\gamma I = -j\omega CV + \gamma i$$

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$$

$$-\gamma V = -j\omega LI$$

Week 14

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TWT

$$V = -\frac{\gamma\gamma_0 Z_0}{\gamma^2 - \gamma_0^2} i \qquad E_1 = -\frac{\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} i$$

Circuit equation

$$(\gamma^2 - \gamma_0^2)(j\beta_e - \gamma)^2 = -j\frac{\gamma^2 \gamma_0 Z_0 \beta_e I_0}{2V_o}$$

TWT

$$\gamma_1 = -\beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_2 = \beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_3 = j\beta_e(1 - C)$$

$$\gamma_4 = -j\beta_e\left(1 - \frac{C^3}{4}\right)$$

$$C \equiv \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$$

C is TWT gain parameter

The output voltage is

$$V(\ell) \approx \frac{V(0)}{3} \exp\left(\frac{\sqrt{3}}{2}\beta_e C \ell\right) \exp\left[-j\beta_e\left(1 + \frac{C}{2}\right)\ell\right]$$

With

$$N = \frac{\ell}{\lambda_e} \quad \text{and} \quad \beta_e = \frac{2\pi}{\lambda_e}$$

the magnitude of $V(\ell)$ is

$$V(\ell) = \frac{V(0)}{3} \exp\left(\sqrt{3}\,\pi NC\right)$$

The output power gain in decibels is defined as

$$A_p \equiv 10 \log\left|\frac{V(\ell)}{V(0)}\right|^2 = -9.54 + 47.3NC \qquad \text{dB}$$

TWT

A helix TWT operates at 4 GHz under a beam voltage 10 kV and beam current 500 mA. If the helix impedance is 25 Ω and the interaction length is 20 cm, find the output power gain in dB.

Solution: $v_0 = 0.593\times10^6 \sqrt{V_0} = 0.593\times10^8$ m/sec .

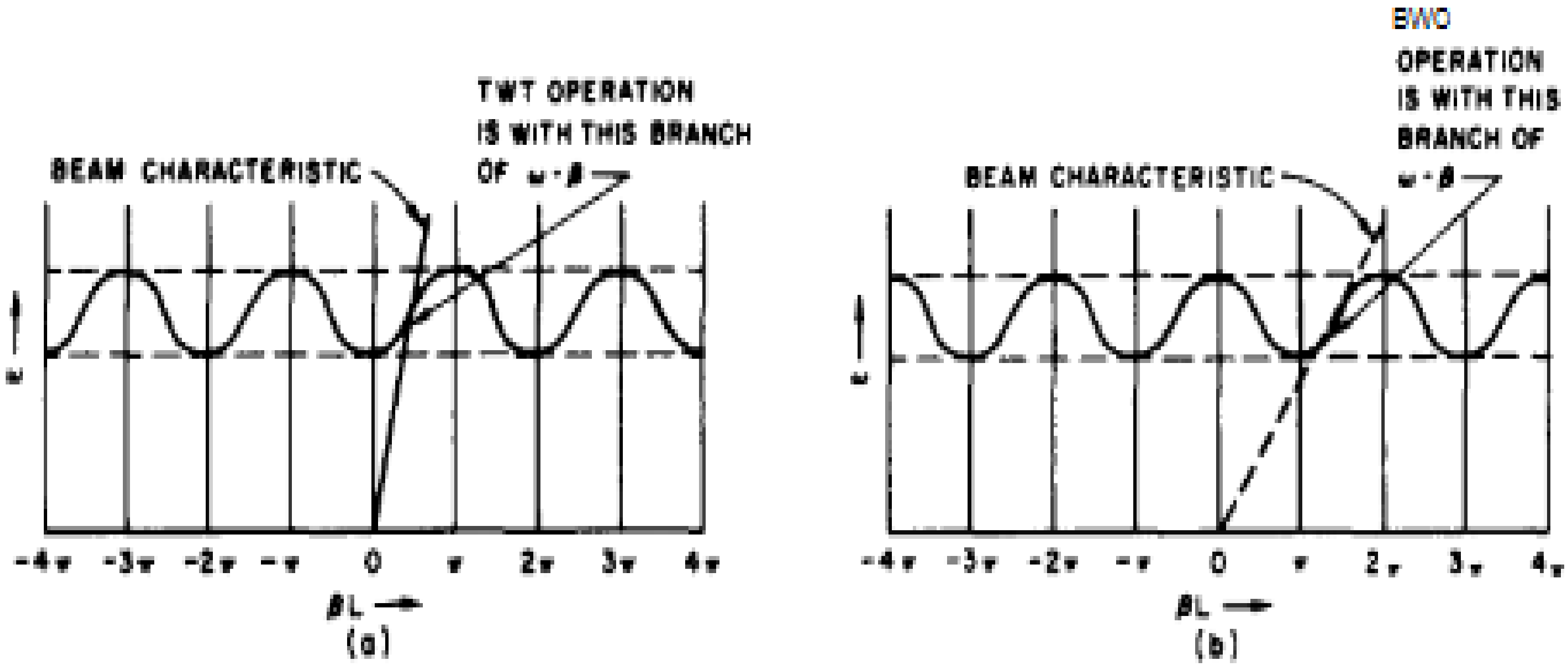
N$= \ell/\lambda_e = \ell\omega/(2\pi v_0) = 13.49$

$$C \equiv \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3} = 0.068$$

$$A_p \equiv 10 \log \left|\frac{V(\ell)}{V(0)}\right|^2 = -9.54 + 47.3NC \quad \text{dB} = 33.85 \text{ dB}$$

BWO

BWO is similar to TWT. The only difference in its operation. In TWT beam interacts with the forward going wave in SWS; however, in BWO beam interacts with the backward going wave.



MAGNETRON

The magnetron is a high-powered vacuum tube that generates microwaves using the interaction of a stream of electrons with a crossed electric and magnetic field.

High-power oscillator

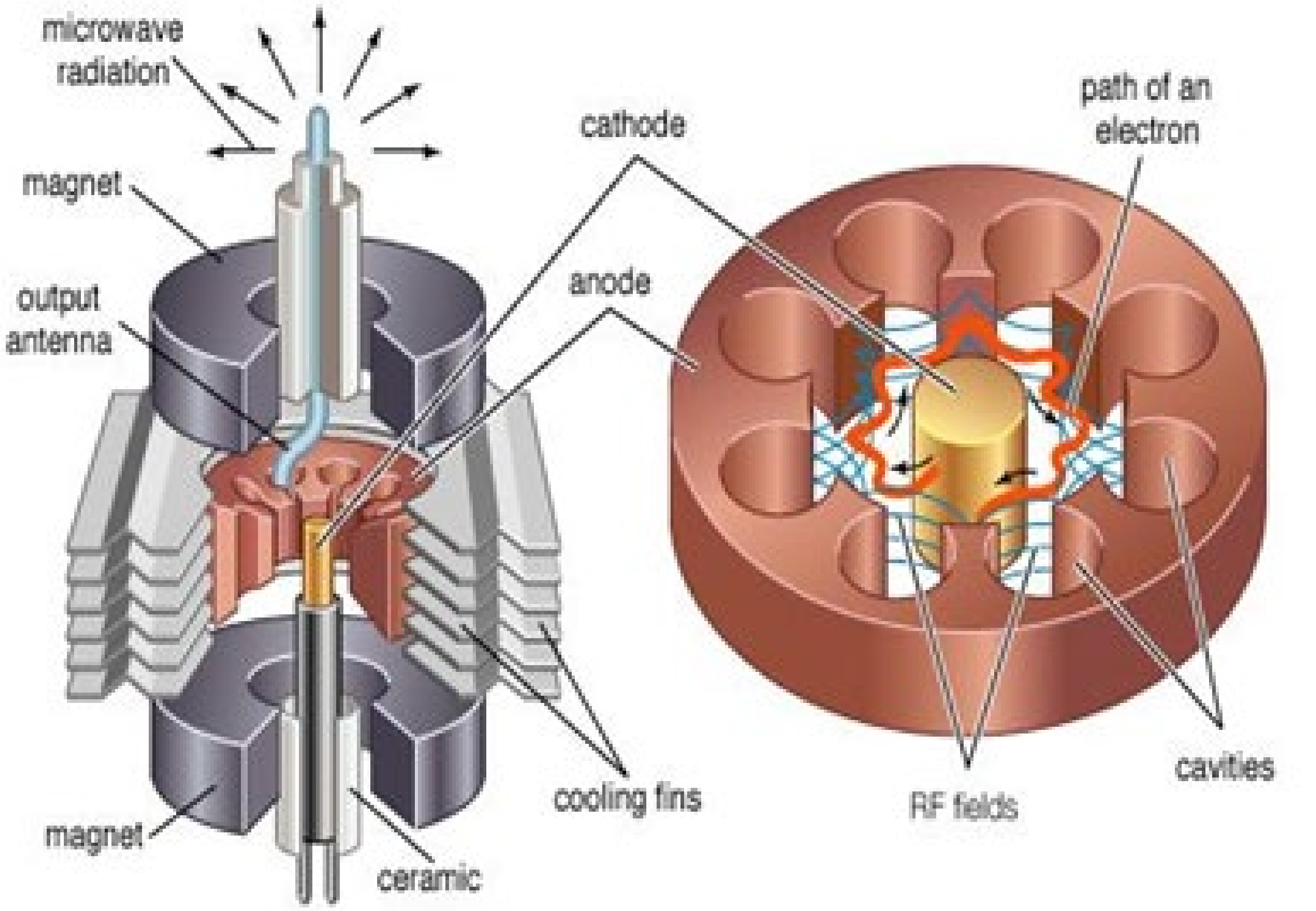
Common in radar and microwave ovens

Cathode in center, anode around outside

Strong dc magnetic field around tube causes electrons from cathode to spiral as they move toward anode

Current of electrons generates microwaves in cavities around outside

MAGNETRON



microwave radiation
magnet
output antenna
cathode
anode
path of an electron
cooling fins
magnet
ceramic
RF fields
cavities

Electron dynamics in magnetic field

The Lorentz force on an electron in magnetic field:

$$\vec{F} = -e\vec{v} \times \vec{B}$$

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left(B_z\frac{dy}{dt} - B_y\frac{dz}{dt}\right)$$

$$\frac{d^2y}{dt^2} = -\frac{e}{m}\left(B_x\frac{dz}{dt} - B_z\frac{dx}{dt}\right)$$

$$\frac{d^2z}{dt^2} = -\frac{e}{m}\left(B_y\frac{dx}{dt} - B_x\frac{dy}{dt}\right)$$

DC magnetic field $\vec{B} = B_0\vec{a}_z$; Initial velocity $\vec{v}(t=0) = v_0\vec{a}_x$.

$$v_x = v_0 \cos\omega_c t$$

$$v_y = v_0 \sin\omega_c t$$

$$\omega_c = \frac{eB_0}{m}$$

Electron dynamics in magnetic field

$$x = \frac{v_0}{\omega_c} \sin \omega_c t$$

$$y = -\frac{v_0}{\omega_c} \cos \omega_c t$$

$$x^2 + y^2 = \left(\frac{v_0}{\omega_c} \right)^2$$

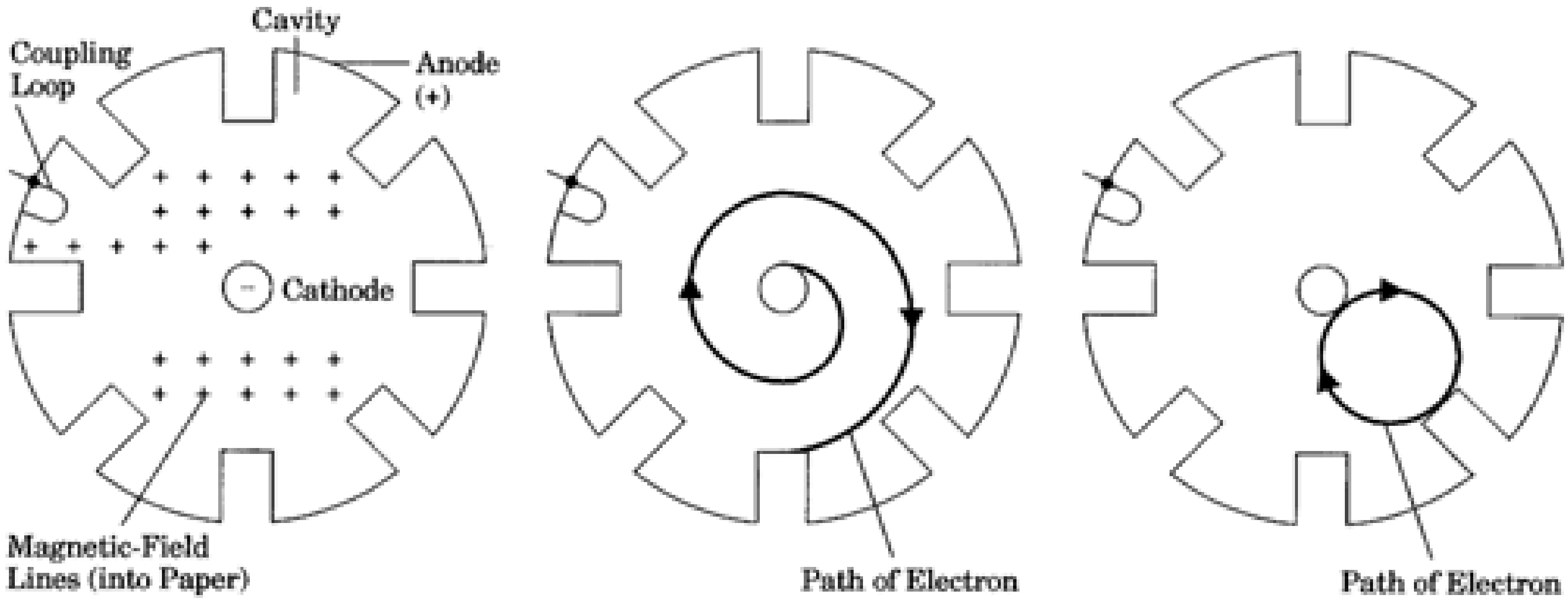
$$R = \frac{v_0}{\omega_c} = \frac{m v_0}{e B_0}.$$

Gyro-radius/Cyclotron radius

Week 15

Slide 261-282

MAGNETRON

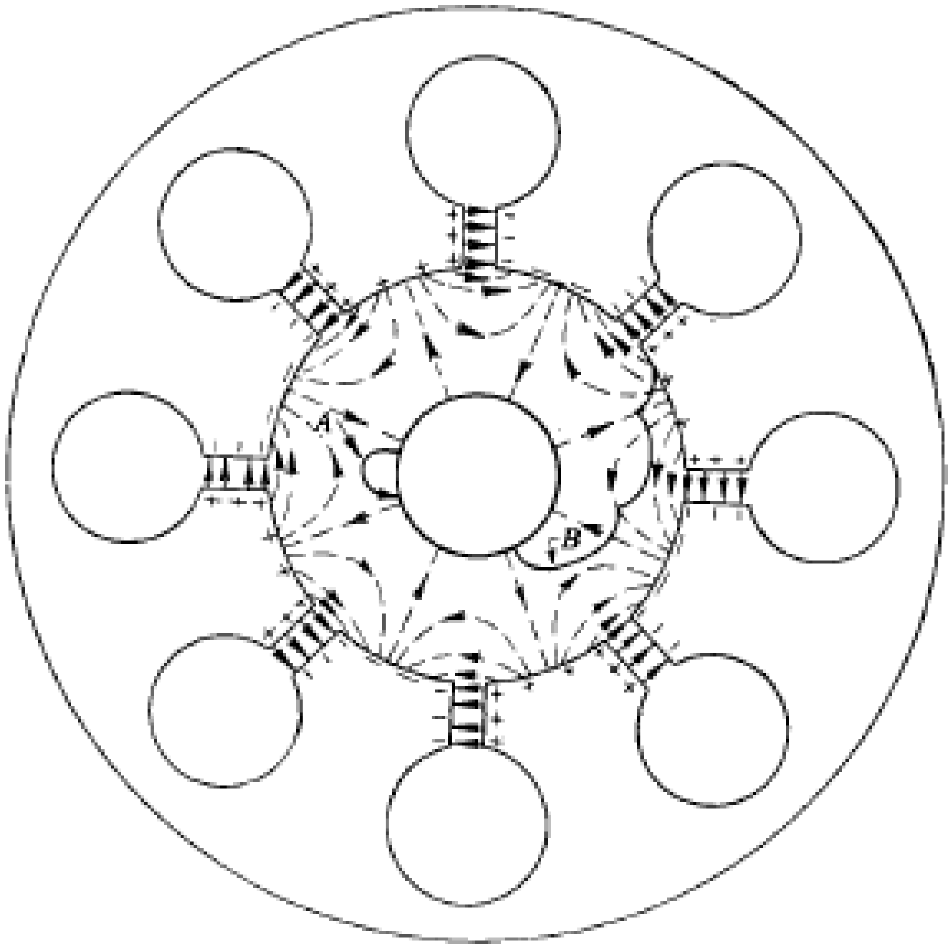


(a) Cross Section

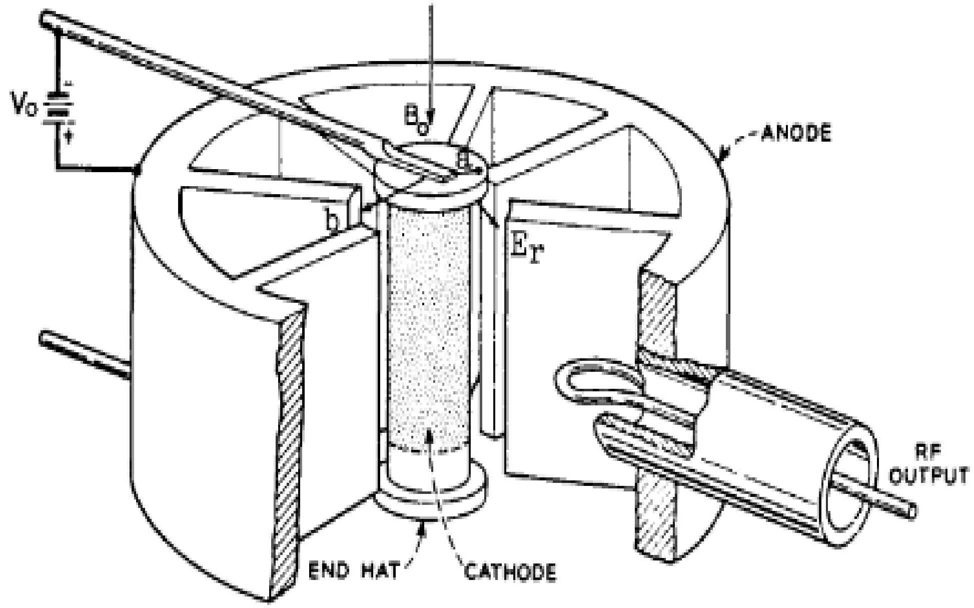
(b) Electron Paths in Normal Operation

(c) Electron Paths at Cutoff

Electron motion in magnetron



Electron motion in magnetron



$$\vec{E} = -E_\rho \vec{a}_\rho$$

$$\vec{B} = B_0 \vec{a}_z$$

$$\frac{d^2\rho}{dt^2} - \rho\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_\rho - \frac{e}{m}B_0\rho\frac{d\phi}{dt}$$

$$\frac{1}{\rho}\frac{d}{dt}\left(\rho^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_0\frac{d\rho}{dt}$$

Boundary condition: $d\phi / dt = 0 \quad at \;\; \rho = a$

Electron motion in magnetron

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{\rho^2}\right)$$

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = v_\rho^2 + v_\phi^2 = \left(\frac{d\rho}{dt}\right)^2 + \left(\rho\frac{d\phi}{dt}\right)^2$$

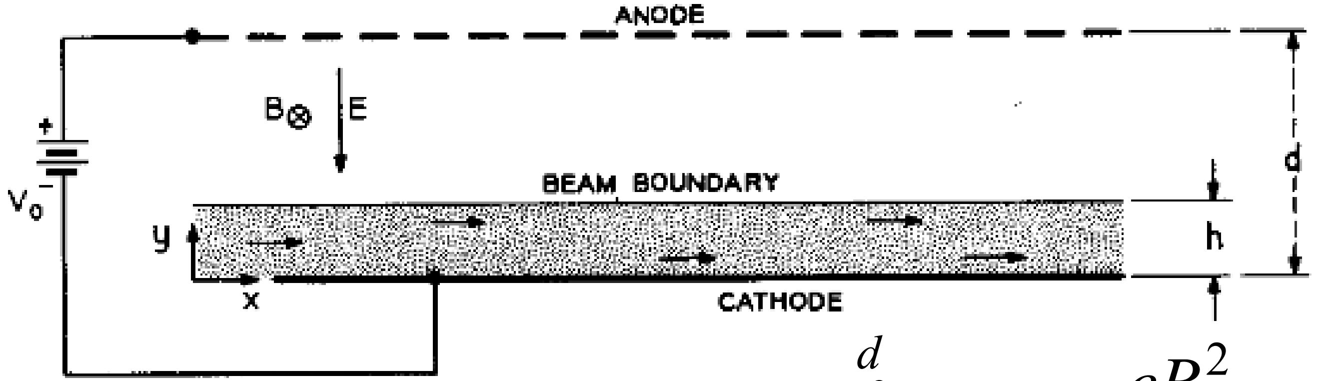
Boundary condition: $d\rho / dt = 0 \quad and\ V = V_0 \ \ at\ \ \rho = b$

$$B_{0c} = \frac{\left(8V_0\dfrac{m}{e}\right)^{1/2}}{b\left(1 - \dfrac{a^2}{b^2}\right)}$$

Hull cutoff condition

$$V_{0c} = \frac{e}{8m}B_0^2 b^2\left(1 - \frac{a^2}{b^2}\right)^2$$

Linear magnetron



$$v_x = \frac{E(y)}{B} = \frac{1}{B}\frac{dV}{dy}$$

$$v_y = v_z = 0.$$

$$V = \frac{eB^2}{2m} y^2$$

$$E_y = -\frac{dV}{dy} = -\frac{eB^2}{m} h$$

$$V_0 = -\int_0^d E_y dy = \frac{eB^2}{m} h(d - h/2)$$

$$V_{0c} = \frac{eB^2}{2m} d^2$$

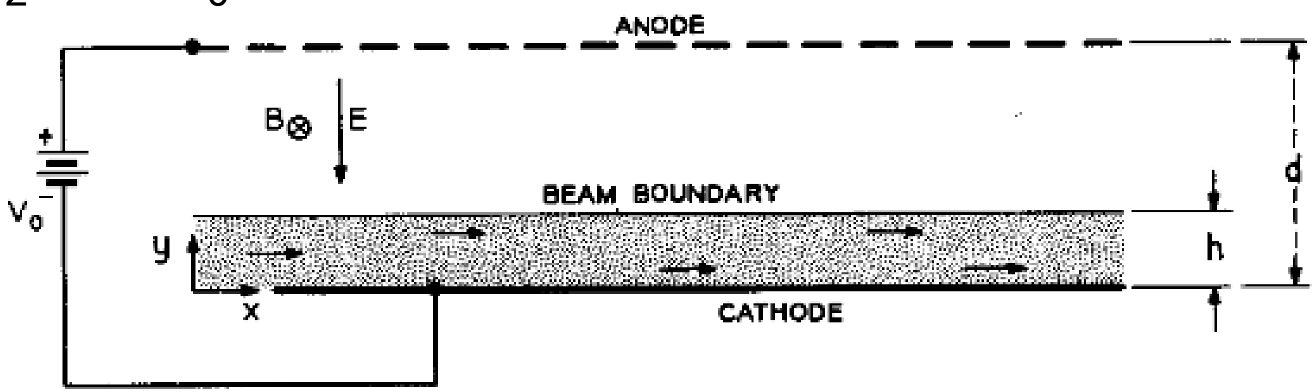
$$B_{0c} = \frac{1}{d}\sqrt{2\frac{m}{e}V_0}$$

Hull cutoff condition

Linear magnetron

<u>The Hartree condition:</u>

The Hull cutoff equations derived so far are in absence of electromagnetic wave. However, if interaction between beam and RF wave is to take place, the beam parallel to the circuit must have phase velocity approximately equal to the phase velocity of the wave. This leads to a relation between B_z and V_0 known as Hartree condition.



The Hartree condition of Linear magnetron

$$V = \frac{eB^2}{2m} y^2$$

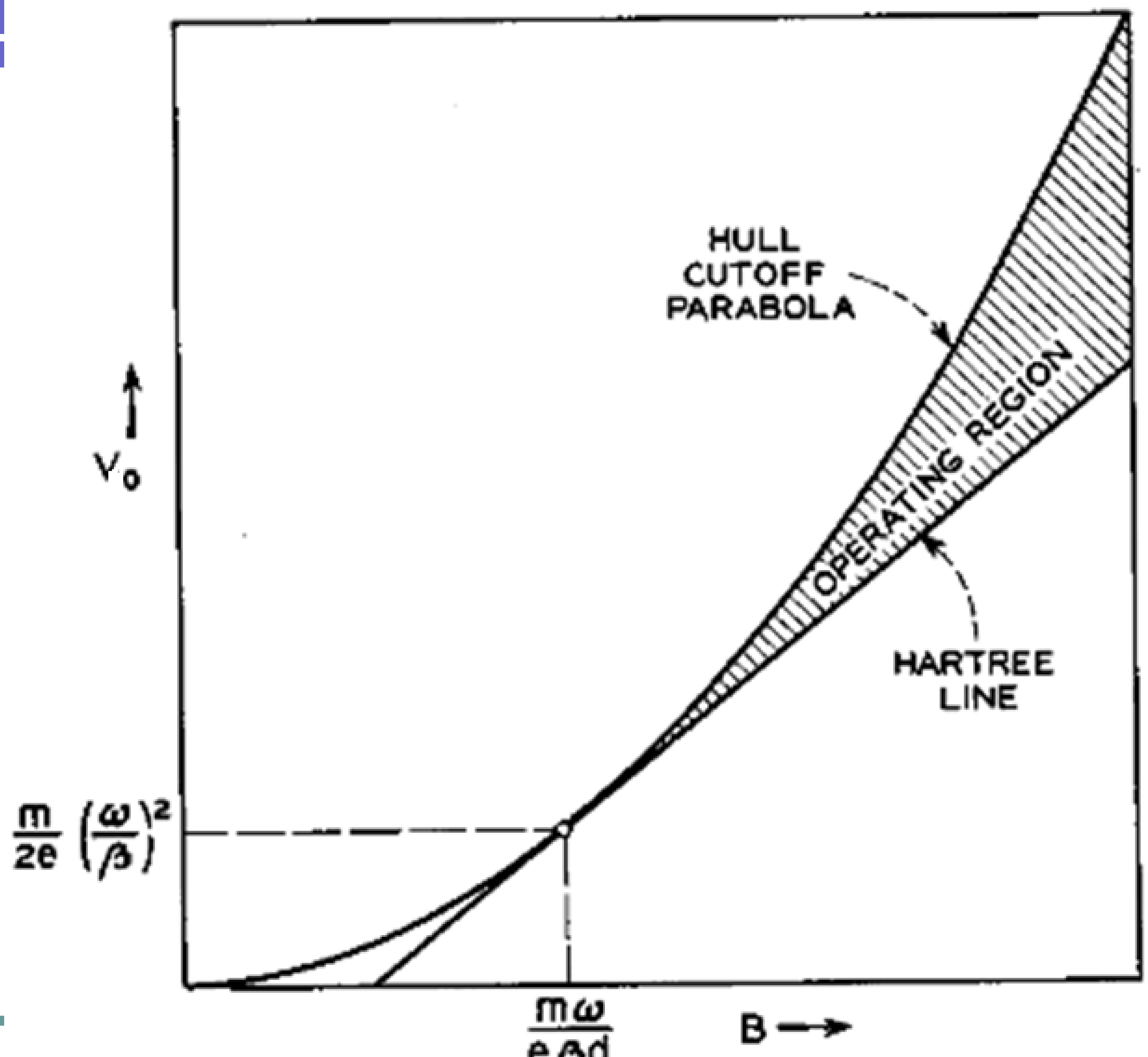
$$v_x(h) = \frac{E(y)}{B} = \frac{1}{B}\frac{dV}{dy} = \frac{eB}{m} h$$

$$\frac{\omega}{\beta} = \frac{eB}{m} h$$

$$\boxed{V_{0h} = \frac{\omega B_0 d}{\beta} - \frac{m}{2e}\frac{\omega^2}{\beta^2}}$$

Hartree condition

Operating region of Linear magnetron



EEE 4181

MICROWAVE ENGINEERING

PART 5:

Microwave Semiconductor Devices

Solid state devices

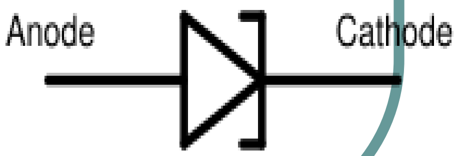
In most of the low power applications, solid state devices have replaced electron beam devices because of the advantages of their small size, light weight, high reliability, low cost and capability of being incorporated into microwave integrated circuits.

Solid state devices

- Microwave solid-state devices
  - Microwave transistor
    - Microwave BJT
    - HBT
    - ~~Tunnel diode~~
  - Field-effect transistors
    - JFET
    - MESFET, HEMT
    - MOSFET
    - NMOS, PMOS, CMOS
    - Memories
    - CCD
  - Transferred electron devices
    - Gunn diode
    - LSA diodes
    - InP diodes
    - CdTe diodes
  - Avalanche transit-time devices
    - Read diode
    - IMPATT diode
    - TRAPATT diode
    - BARITT diode

TUNNEL DIODE (Esaki Diode)

- It was introduced by Leo Esaki in 1958.
- Heavily-doped p-n junction
  - Impurity concentration is 1 part in 10^3 as compared to 1 part in 10^8 in p-n junction diode
- Width of the depletion layer is very small (about 10nm).
- It is generally made up of Ge and GaAs.
- It shows tunneling phenomenon.
- Circuit symbol of tunnel diode is :



WHAT IS TUNNELING

- **Classically**, carrier must have energy at least equal to potential-barrier height to cross the junction .
- But according to **Quantum mechanics** there is finite probability that it can penetrate through the barrier for a thin width.
- This phenomenon is called **tunneling** and hence the Esaki Diode is know as **Tunnel Diode**.

CHARACTERISTIC OF TUNNEL DIODE

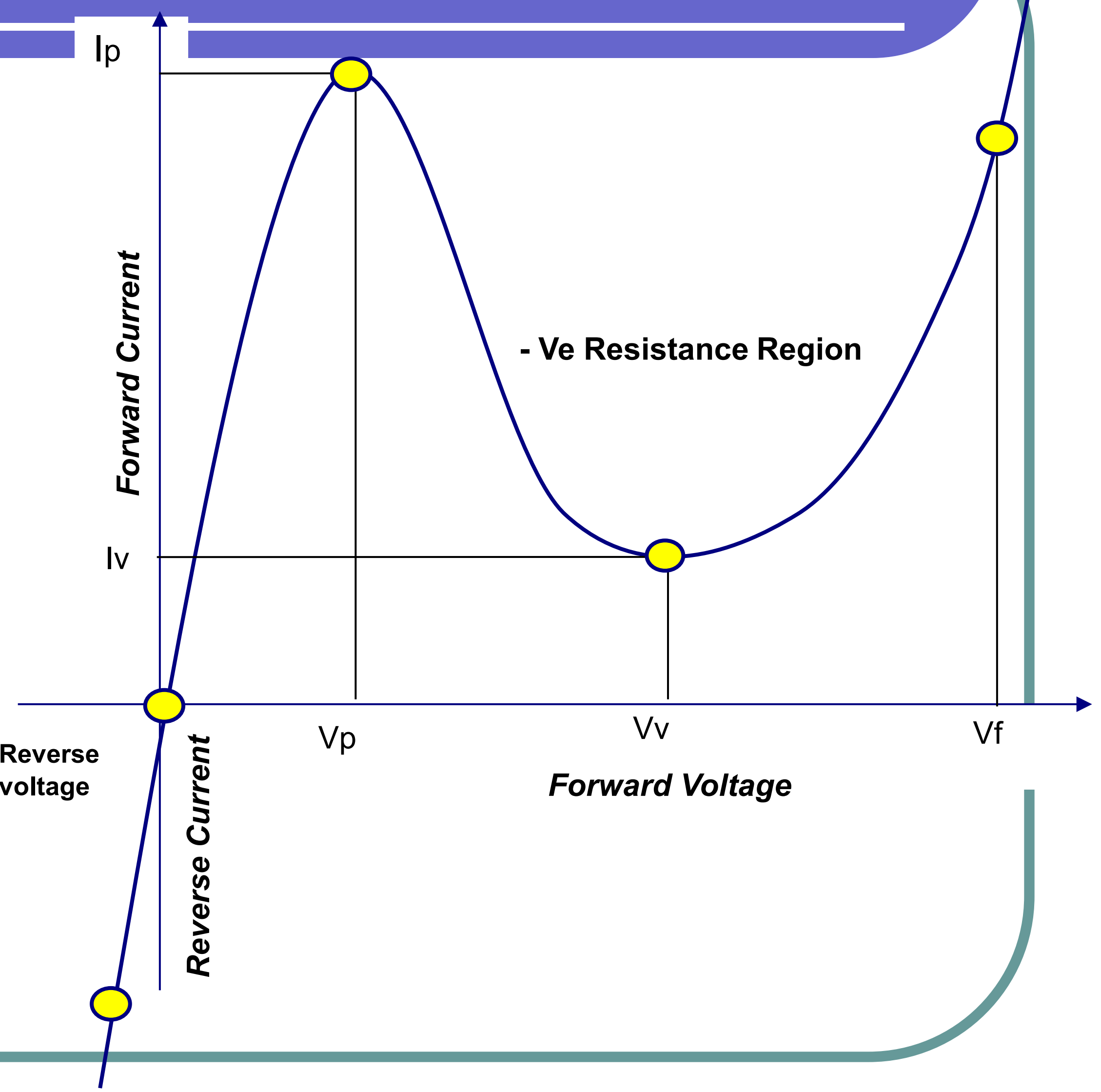
Ip:- Peak Current

Iv :- Valley Current

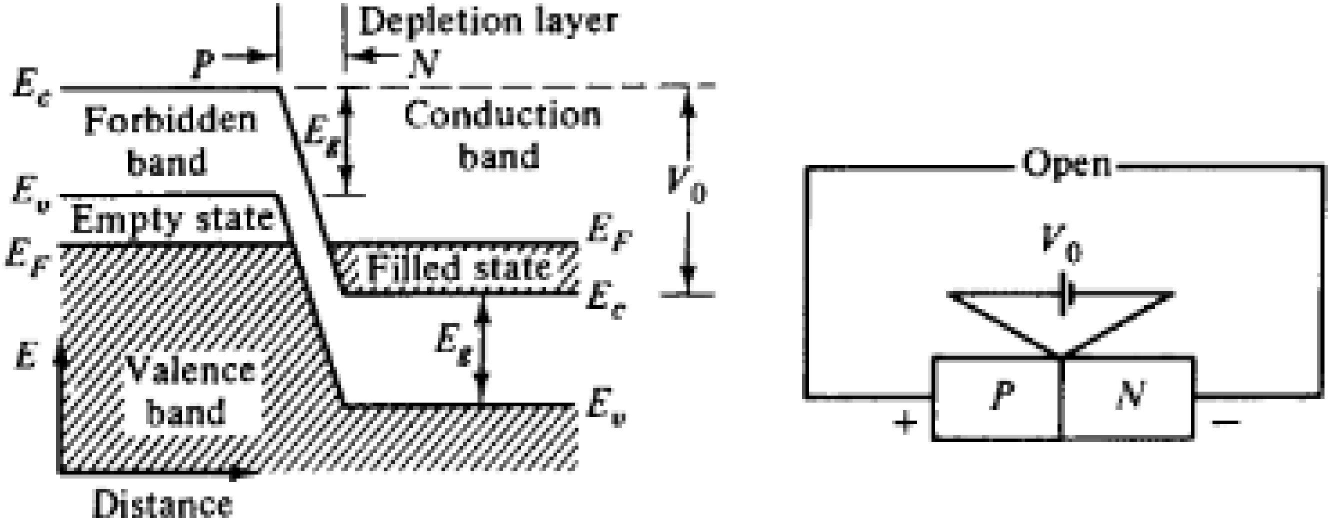
Vp:- Peak Voltage

Vv:- Valley Voltage

Vf:- Peak Forward Voltage



ZERO BIAS



(a) Tunnel diode under zero-bias equilibrium

- -Zero current on the I-V diagram;
- -All energy states are filled below E_F on both sides of the junction;

E_F is the Fermi level representing the energy state with 50% probability of being filled if no forbidden band exists

V_0 is the potential barrier of the jur

E_g is the energy required to break a … h is 0.72 eV for germanium and 1.10

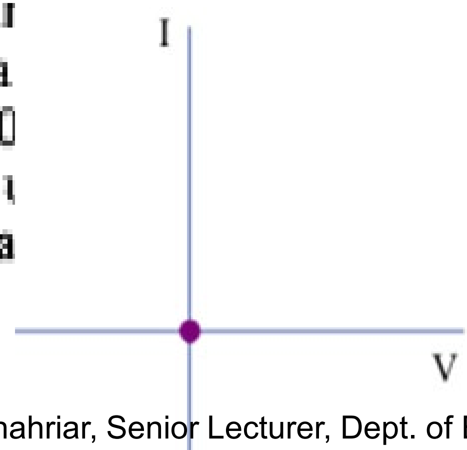
E_c is the lowest energy in the condu

E_v is the maximum energy in the va

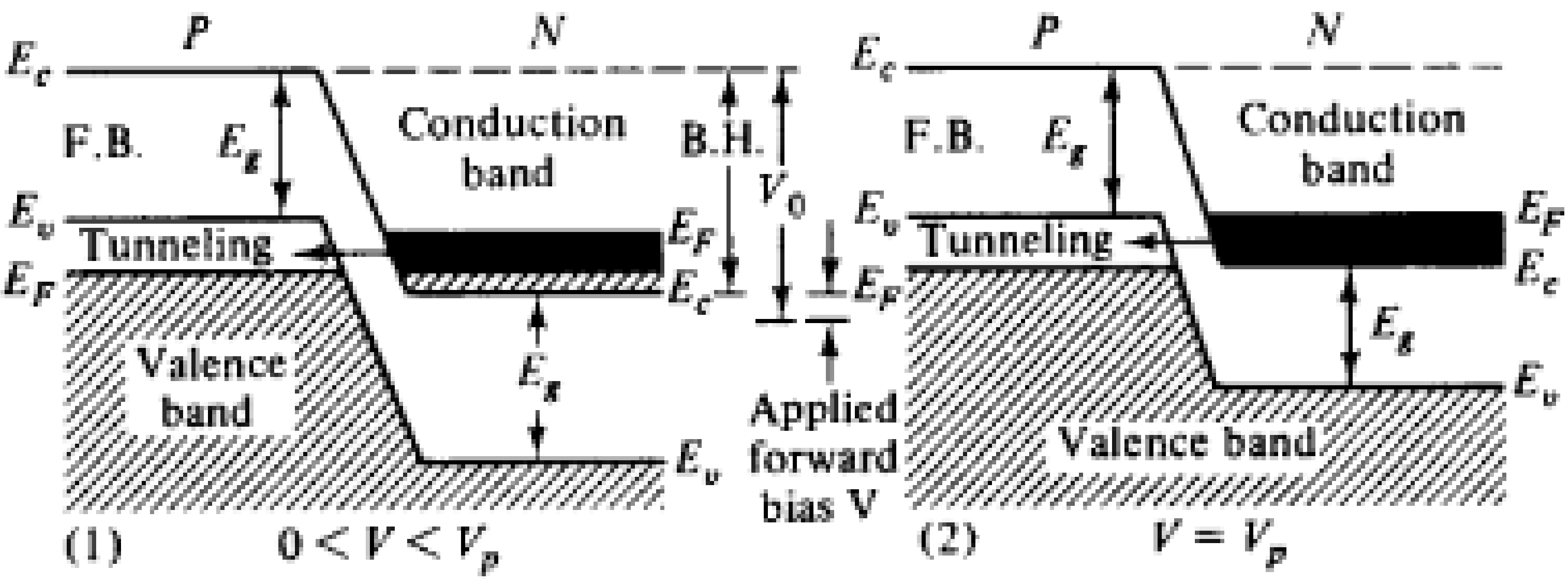
V is the applied forward bias

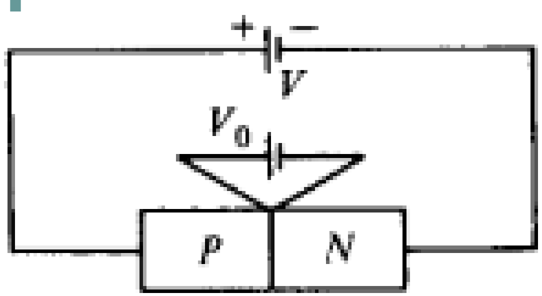
F.B. stands for the forbidden band

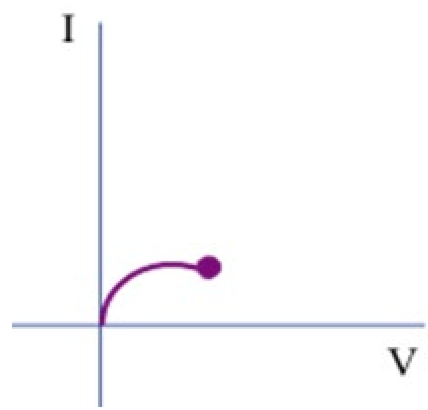
B.H. represents the barrier height



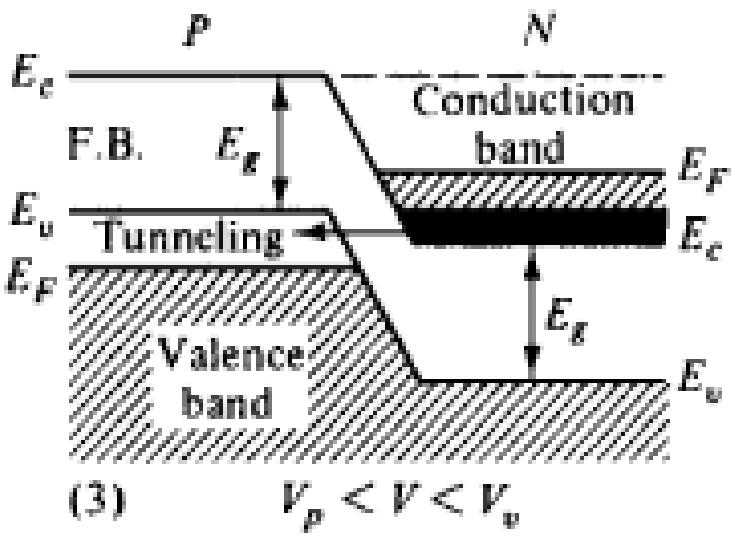
FORWARD BIAS-1

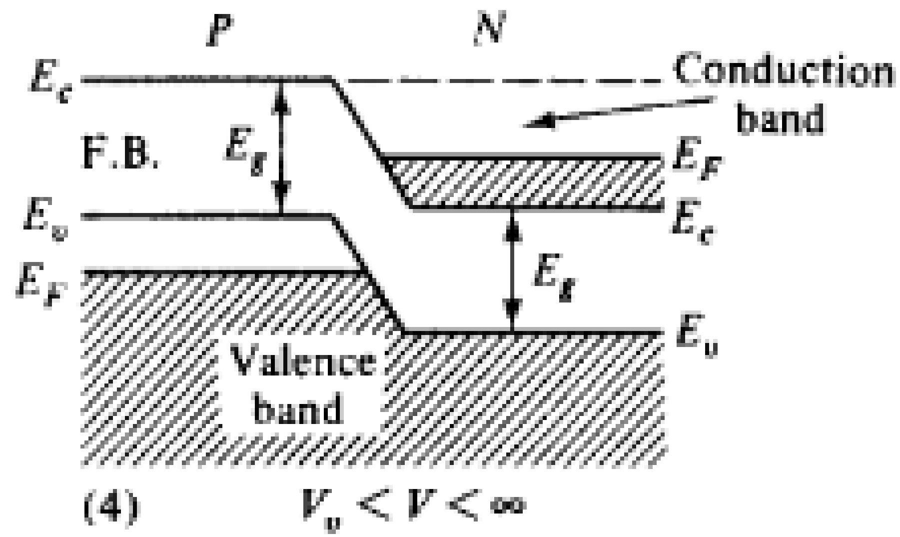


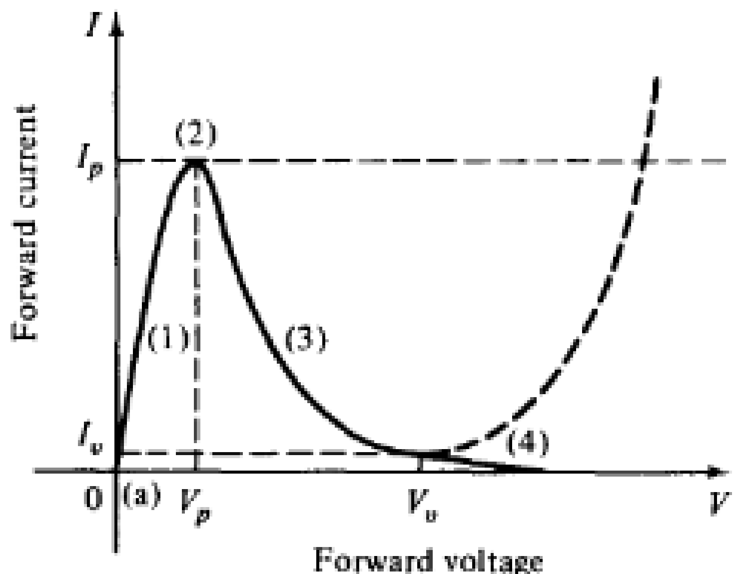




FORWARD BIAS-2

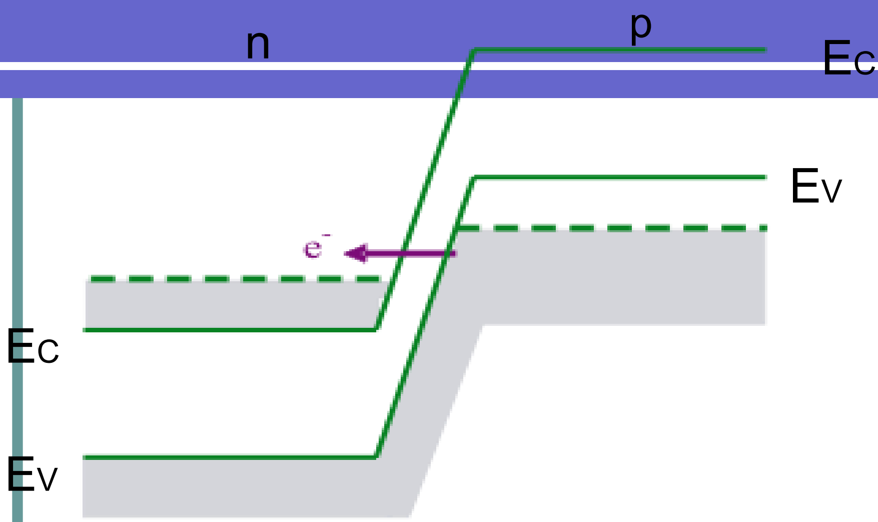




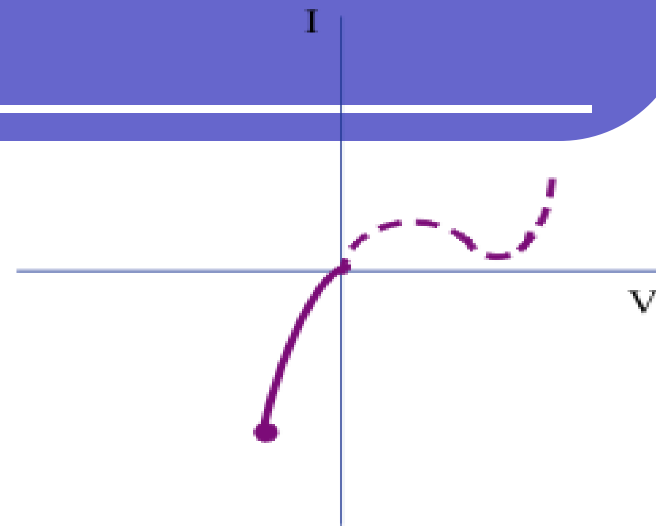


(a) Tunnelling current (solid curve) and injection current (dashed curve)

REVERSE BIAS



A simplified energy-band diagram of a tunnel diode with a reverse bias voltage



I-V characteristic of a tunnel diode with a reverse-bias voltage.

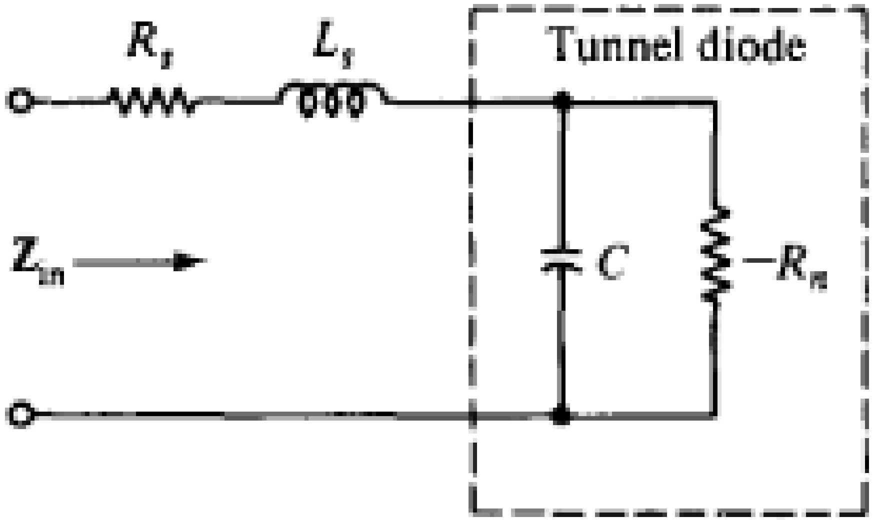
- Electrons in the valence band on the p side are directly opposite to empty states in the conduction band on the n side.

-Electrons tunnel directly from the p region into the n region.

- The reverse-bias current increases monotonically and rapidly with reverse-bias voltage.

TUNNEL DIODE EQUIVALENT CIRCUIT

- This is the equivalent circuit of tunnel diode when biased in negative resistance region.



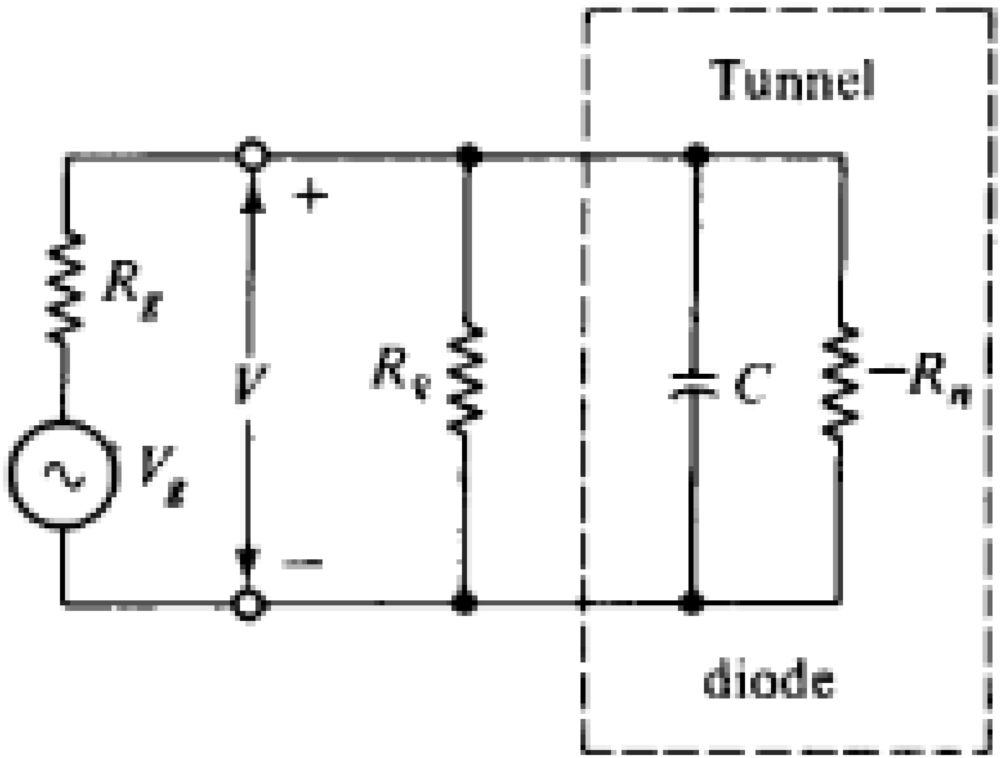
- At higher frequencies the series R_s and L_s can be ignored.
- Hence equivalent circuit reduces to parallel combination of junction capacitance and negative resistance.

$$\mathbf{Z}_{in} = R_s + j\omega L_s + \frac{R_n[\,j/(\omega C)]}{-R_n - j/(\omega C)}$$

$$\mathbf{Z}_{in} = R_s - \frac{R_n}{1 + (\omega R_n C)^2} + j\left[\omega L_s - \frac{\omega R_n^2 C}{1 + (\omega R_n C)^2}\right]$$

$$f_c = \frac{1}{2\pi R_n C}\sqrt{\frac{R_n}{R_s} - 1}$$

$$f_r = \frac{1}{2\pi R_n C}\sqrt{\frac{R_n^2 C}{L_s} - 1}$$



(a) Parallel loading

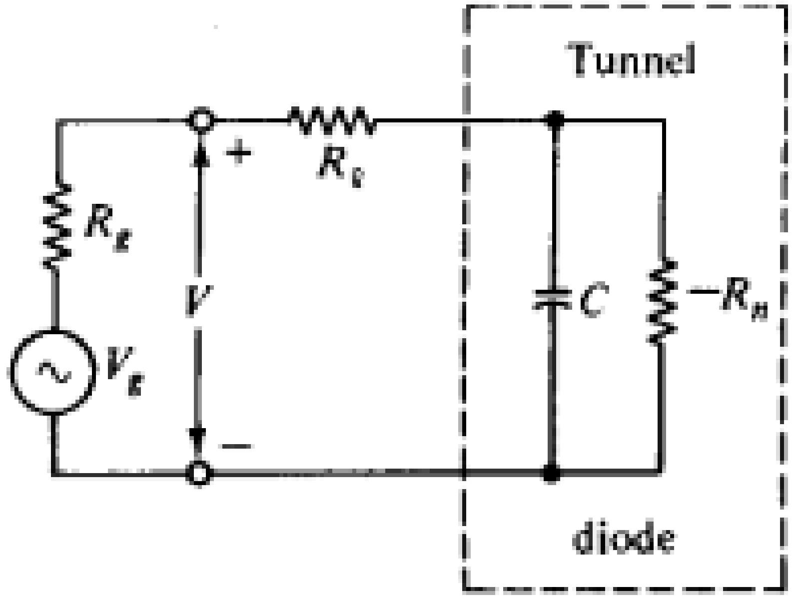
$$P_{out} = \frac{V^2}{R_\ell}$$

$$P_{in} = \frac{V^2}{AR_\ell}$$

$$P_n = \frac{V^2}{R_n}$$

$$\frac{V^2}{AR_\ell} + \frac{V^2}{R_n} = \frac{V^2}{R_\ell}$$

$$A = \frac{R_n}{R_n - R_\ell}$$



(b) Series loading

$$A = \frac{R_\ell}{R_\ell - R_n} = \frac{1}{1 - R_n/R_\ell}$$

Week 17

Slide 284-295

TRANSFERRED ELECTRON DEVICE

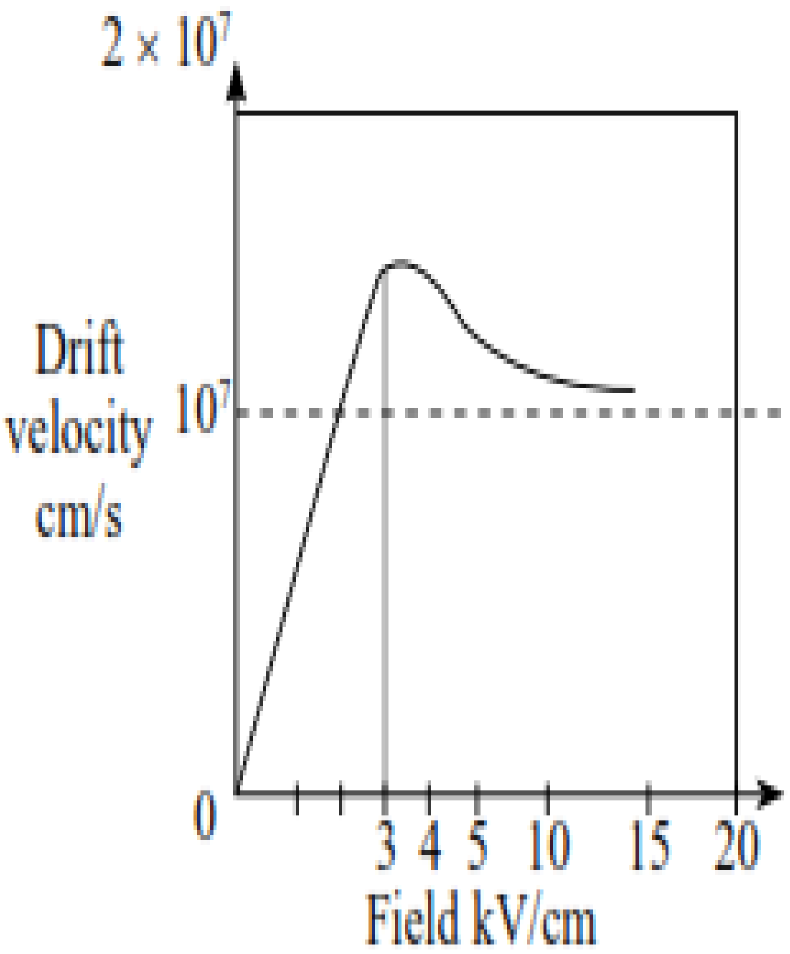
- A **Gunn diode**, also known as a **transferred electron device** (TED).
- Used as HF electronic oscillators in radar speed guns, microwave relay data link transmitters, and automatic door openers.
- It consists only of n-doped semiconductor material.
- Gunn diode has a region of negative differential resistance.
- Gallium Arsenide Gunn diodes are made for frequencies up to 200 GHz whereas Gallium Nitride can reach up to 3 THz.

IMAGE OF GUNN DIODE

TRANSFERRED ELECTRON DEVICE

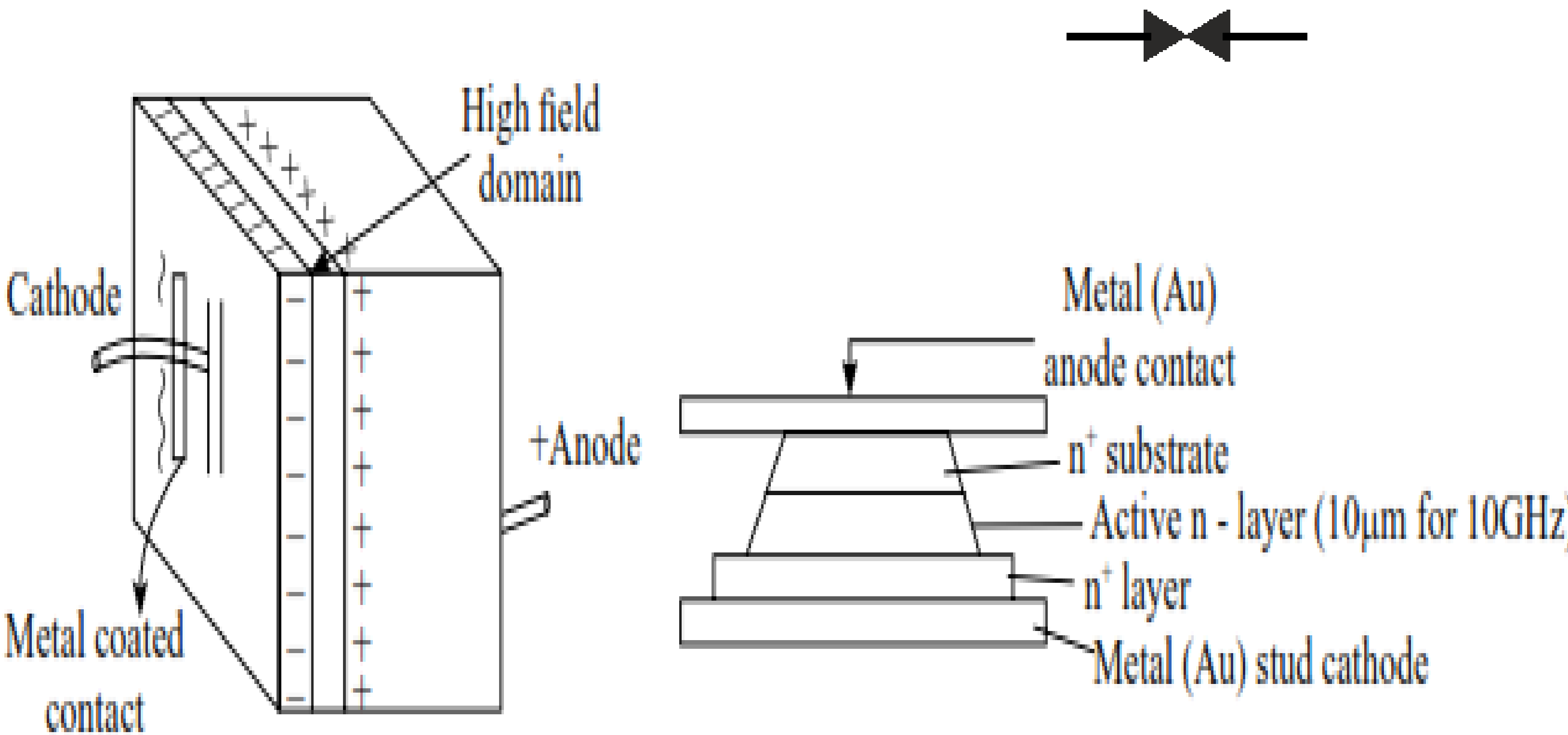
Gunn effect:

From Gunn's observation the carrier drift velocity is linearly increased from zero to a maximum when electric field is varied from zero to a threshold value (3000V/cm for n type GaAs), when E field is beyond the threshold value then the drift velocity decreased and the diode exhibits Negative resistance as shown in figure. This is called Gunn effect.



TRANSFERRED ELECTRON DEVICE

Construction of details of Gunn diode:



Internal construction is unlike other diodes in that it consists only of N-doped semiconductor material.

TRANSFERRED ELECTRON DEVICE

In Gunn diode, three regions exist: two of those are heavily N-doped on each terminal, with a thin layer of lightly n-doped material between. When a voltage is applied to the device, the electrical gradient will be largest across the thin middle layer. If the voltage is increased, the current through the layer will first increase, but eventually, at higher field values, the conductive properties of the middle layer are altered, increasing its resistivity, and causing the current to fall.

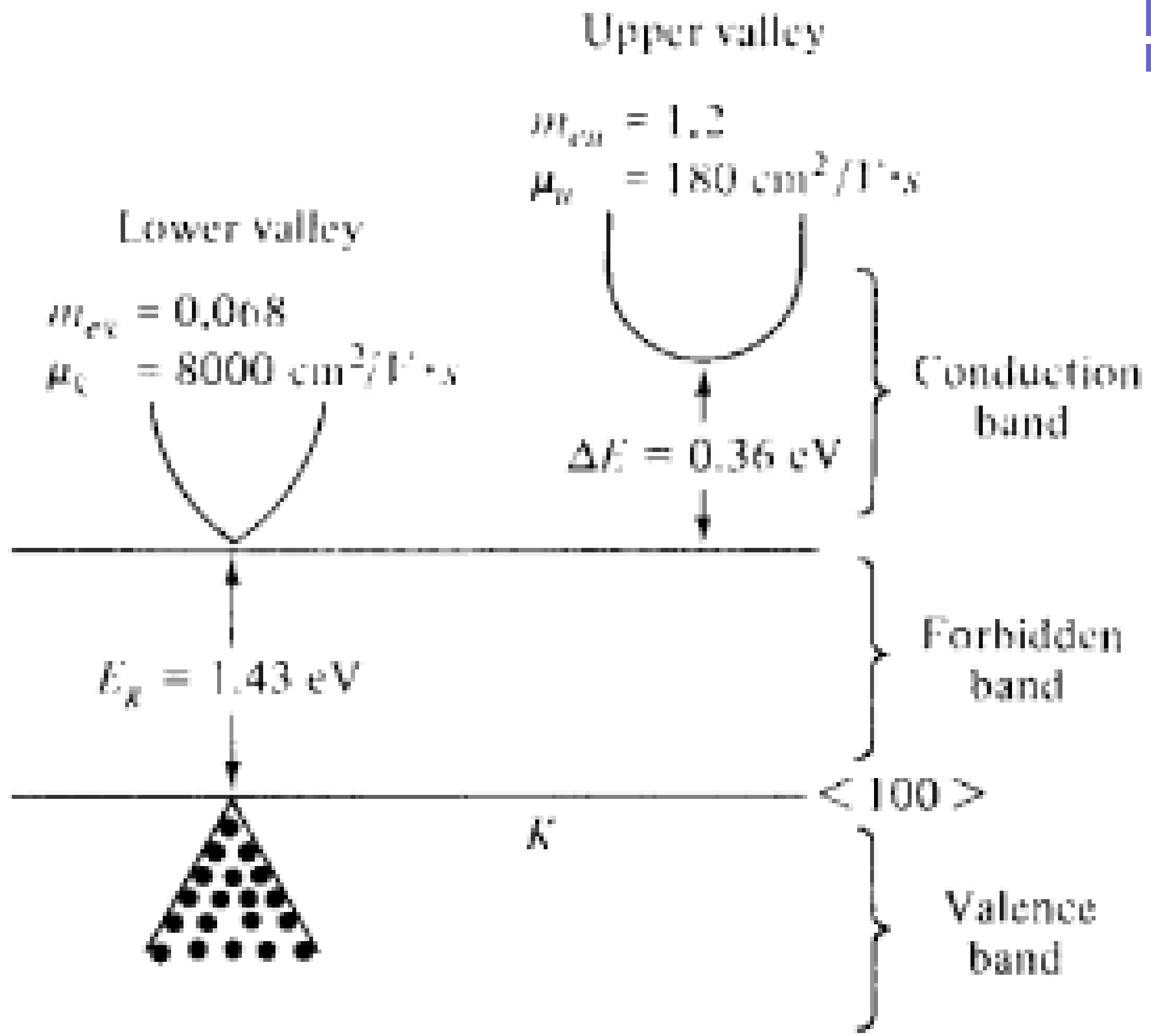
The Gunn diodes are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or cadmium telluride (CdTe).

TRANSFERRED ELECTRON DEVICE

Ridley-Watkins-Hilsum theory for semiconductors to exhibit negative resistance:

1. The separation energy between the bottom of the lower valley and the bottom of the upper valley must be several times larger than the thermal energy (about 0.026 eV) at room temperature. This means that $\Delta E > kT$ or $\Delta E > 0.026$ eV.
2. The separation energy between the valleys must be smaller than the gap energy between the conduction and valence bands. This means that $\Delta E < E_g$. Otherwise the semiconductor will break down and become highly conductive before the electrons begin to transfer to the upper valleys because hole-electron pair formation is created.
3. Electrons in the lower valley must have high mobility, small effective mass, and a low density of state, whereas those in the upper valley must have low mobility, large effective mass, and a high density of state. In other words, electron velocities (dE/dk) must be much larger in the lower valleys than in the upper valleys.

TRANSFERRED ELECTRON DEVICE

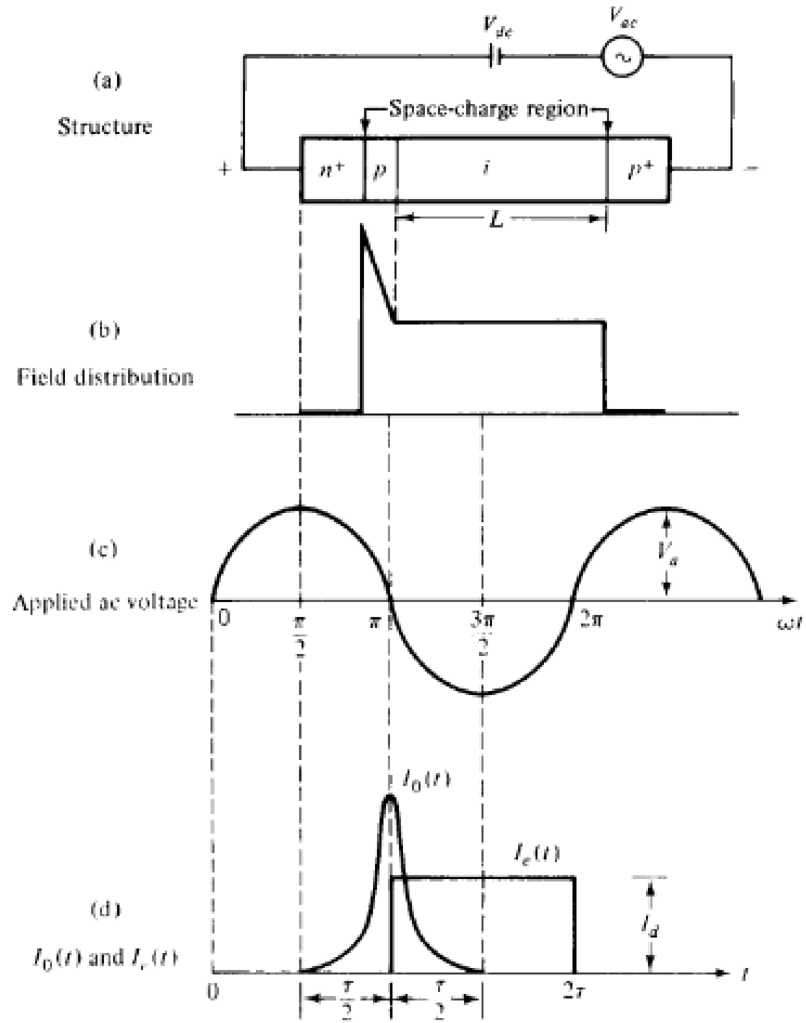


IMPATT DIODE

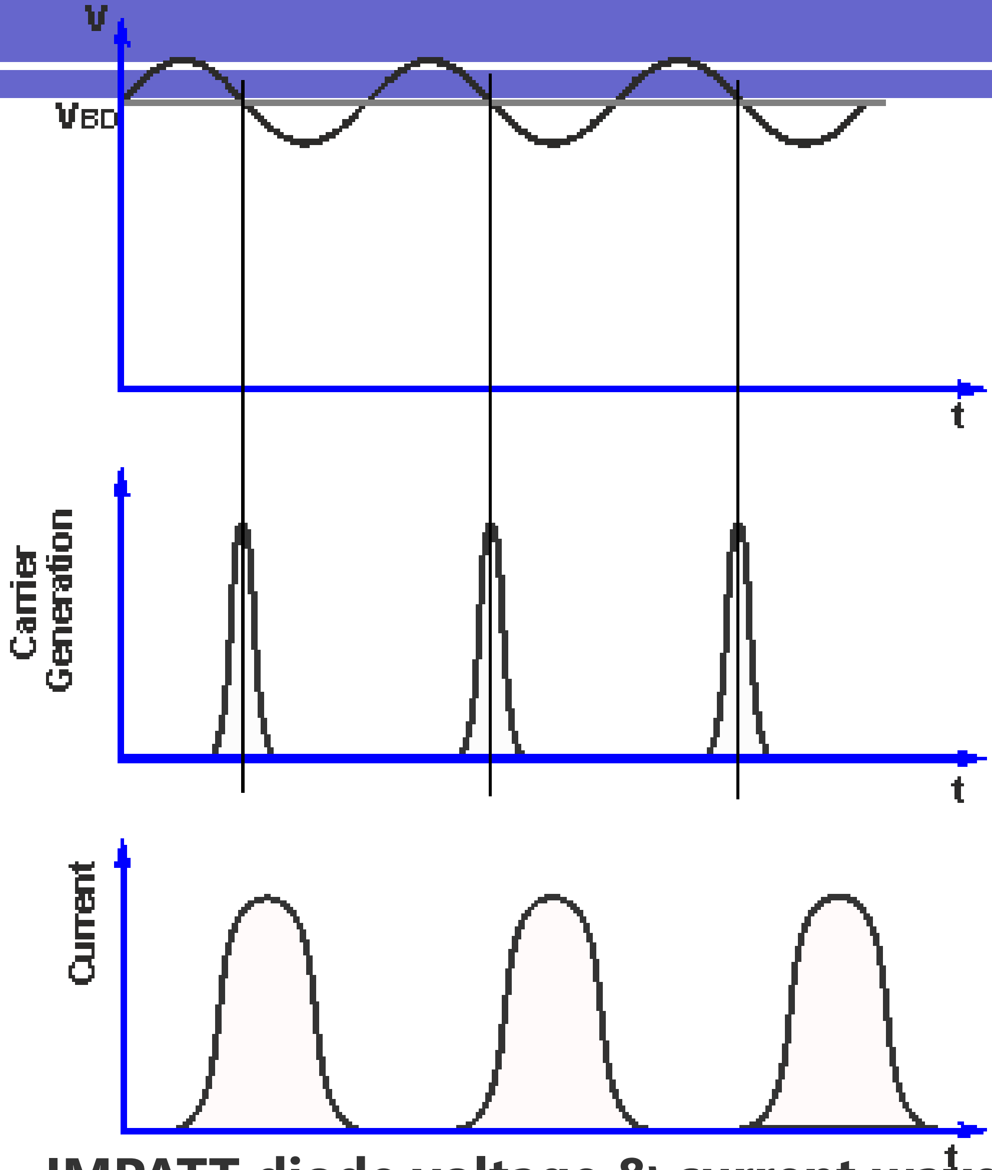
- Impact Avalanche and Transit-Time (IMPATT) Diode, also called Avalanche transit-time diodes
- Multilayer diodes of several different types used to generate microwave power
- In contrast to other devices in this class (tunnel diodes, thyristors, and Gunn diodes), the negative resistance of avalanche-and-transit time diodes appears only at superhigh frequencies.
- Avalanche-and-transit time diodes are used to generate oscillations in the frequency range from 1 to 300 gigahertz

IMPATT DIODE

IMPATT diode is made of n^+-p -i-p^+ or p^+-n -i-n^+ structure. First version of this diode is called READ diode.



IMPATT DIODE



IMPATT diode voltage & current waveforms

IMPATT DIODE

IMPATT diodes exhibits negative resistance by two effects:

- The impact ionization effect, which causes the carrier current $I_0(t)$ and the ac voltage to be out of phase by 90^0
- The transit-time effect, which further delay the external current $I_e(t)$ relative to the ac voltage by 90^0

IMPATT DIODE

IMPATT diodes Application:

- ✓ Used it the final power stage of solid state microwave transmitters for communication purpose
- ✓ Used in TV transmitter
- ✓ Used in FDM/TDM system
- ✓ Used as microwave source in Lab